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The Winner's Edge

The Joy of Hand Calcs

Over the last couple of months I've been recruiting staff for ChassisSim. I've been recruiting from final year under graduates to early post grads. I've also been keeping a close eye on various forums such as the FSAE forum. While the enthusiasm of these engineering students has been beyond question, I have become increasingly concerned about the lack of basic skills I've been seeing. In particular what I am alarmed at has been the lack of many of these senior under grads and recent post grads to do basic hand calculations. This is what we'll be discussing in this article.

The reason I'm writing this article is that if you are serious about engineering the ability to do basic hand calcs is a must have skill. The reason this is a must have skill is the ability to hand calc something (i.e the ability to work it out without a computer) is your ultimate fact filter as well as sanity checking what a computer tells you. This ability has saved my neck on more occasions then I care to remember. Also if you still don't think this is important, the great Kelly Johnson, arguably one of the greatest Aeronautical Engineers who ever lived had this freaky ability to look at an aero surface and could tell you what the pressure distribution, C_L and C_D was to within about 10%. It's no wonder he was able to lead teams that could pull off projects such as the U2 and the SR-71 Blackbird. So if you're serious listen up.

The first hand calc I want to talk about is calculating an RPM limit given a gear ratio and a required pit lane speed limit. I actually had to teach this to a bunch of undergrad students at a V8 Supercar support series race. Let's walk through the parameters they had and what they had to work out,

Table – 1 – Parameters for pit lane speed limit

Parameter	Value
Gear Ratio (Eng speed/wheel speed)	9.2206
Rolling tyre radius	0.325m
Desired pit lane speed limit	40 km/h
RPM	?

The first bit in this process is to figure out the rotational speed of the tyre. Once we know the rotational speed figuring out the RPM of the tyre is easy and to get the engine speed we multiply this by the gear ratio to get the engine RPM. Let's crunch the numbers

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$$
\begin{aligned}
V_T &= r_t \cdot \omega \\
\omega &= \frac{V_T}{r_t} = \frac{40/3.6}{0.325} = 34.18 rad/s \\
\omega &= 2 \cdot \pi \cdot f_{WHEEL} \\
f_{WHEEL} &= \frac{\omega}{2 \cdot \pi} = \frac{34.18}{2 \cdot \pi} = 5.44 Hz \qquad (1) \\
RPM_{WHEEL} &= 60 \cdot f_{WHEEL} = 5.44 \cdot 60 = 326.47 \\
RPM_{ENG} &= GR \cdot RPM_{WHEEL} \\
&= 3010 RPM
\end{aligned}
$$

So we need to set the RPM limit to 3010 RPM in the pit lane limiter section of the engine management software. For the advanced reader my apologies if this was a bit long winded. However as we can see what we did here was really straight forward.

The next thing I want to talk about is calculating downforce, drag and aero balance. My apologies for those of you how know how to do this. I know have repeated this example on a number of occasions but I am still shocked at the number of people who refuse to do it or don't know how. I'm sorry I know I sound like a broken record but I will continue to repeat this example until I see a marked improvement on people in the greater race engineering community doing this.

To kick things off let's talk about zeroing conventions. The best procedure to do this is to look at the data and zero the dampers on the ground or apply an offset as they are coming out of the pits. A very good procedure to do this is illustrated in Figure -1,

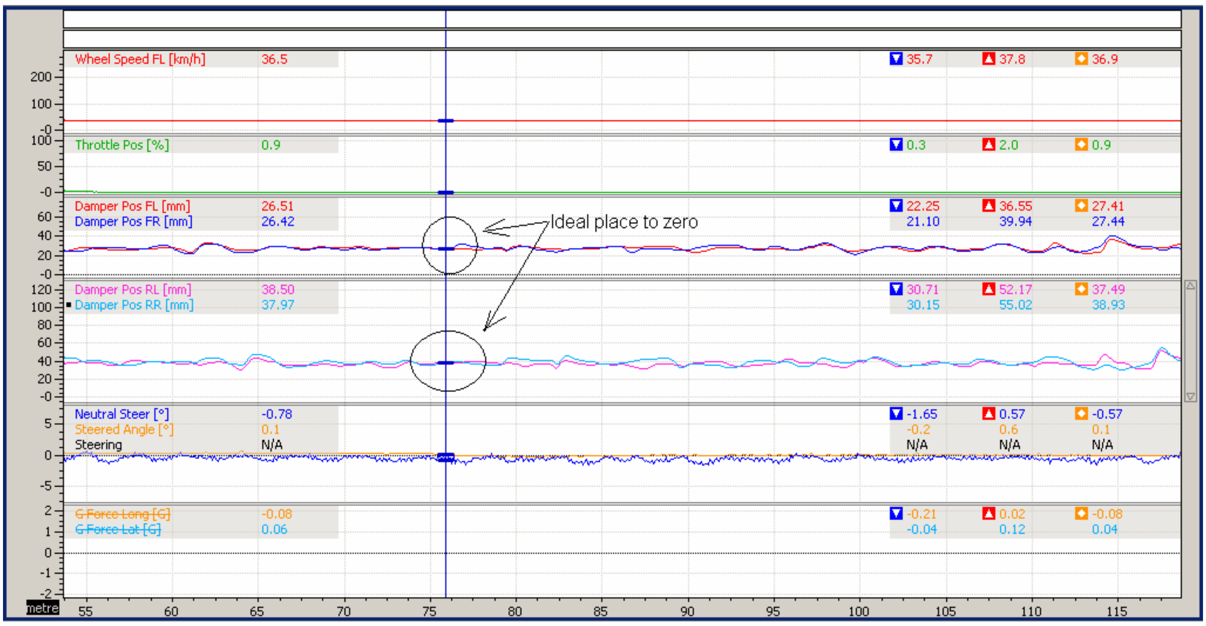

Fig-1 – Ideal place to zero

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As can be seen the dampers are level and this will give us a very good measure of the zero condition.

The next step in the process is to identify where to perform the aero calculation. The best place to take the numbers for this calculation is either the fastest point in the circuit or the longest straight. The thing that takes priority is the car going in a straight line with minimum lateral acceleration. The point to take it from is shown below,

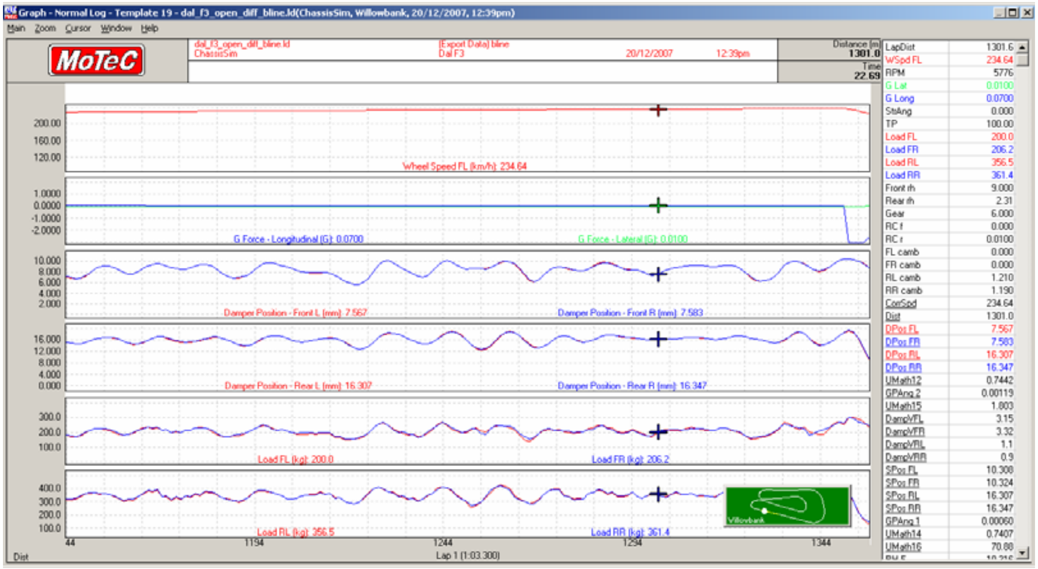

Fig-2 – Where to take hand calculation for aero from.

Also it is wise to filter the data as well. I like to take a low pass freq filter of say 1 Hz. That being said I still like looking at the raw data because it gives me an idea of what the car is doing. Once these points have been established the next point is to calculate the downforce. The best way to illustrate this is by example consider an F3 car,

Table-2 – Sample values for an aero hand calculation.

Item	Quantity
Front Motion Ratio	0.9
Rear Motion Ratio	0.8
FL Damper/FR Damper	10mm/10mm
RL Damper/RR Damper	15mm/15mm
Front spring	140.1 N/mm (800 lbf/in)
Front spring	140.1 N/mm (800 lbf/in)
Torque at RPM	200 Nm
Rolling tyre radius	0.28m
a_x	0g
Vx	220km/h
Gear ratio value	3

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m_t	500kg
h	0.3m
wb	2.6m

Here all motion ratios are Damper on wheel, and the gear ratio is Engine/Wheel velocity and for simplicity I've omitted bump rubbers. Crunching the numbers we see,

$$
\begin{aligned}
FtDownforce &= MR_f \cdot k_f \cdot (FL\_Damp + FR\_Damp) \\
&= 0.9 * 140.1 * (10 + 10) \\
&= 2521.8N \\
RrDownforce &= MR_r \cdot k_r \cdot (RL\_Damp + RR\_Damp) \\
&= 0.8 * 140.1 * (15 + 15) \\
&= 3362.4N \\
C_L A &= \frac{FtDownforce + RrDownforce}{0.5 * 1.225 * (220/3.6)^2} \\
&= 2.57 \\
AeroBal &= 100 \cdot \left(\frac{FtDownforce + \frac{mt \cdot g \cdot a_x \cdot h}{wb}}{FtDownforce + RrDownforce} \right) \\
&= 100 \cdot \left(\frac{2521.8 + \frac{500 \cdot 9.8 \cdot 0 \cdot 0.3}{2.6}}{2521.8 + 3362.4} \right) \\
&= 42.9\% \\
C_D A &= \frac{gr * T / r_t - m_t \cdot g \cdot a_x}{0.5 * 1.225 * (220/3.6)^2} \qquad (2) \\
&= \frac{3 * 200 / 0.28 - 550 \cdot 9.8 \cdot 0}{0.5 * 1.225 * (220/3.6)^2} \\
&= 0.937
\end{aligned}
$$

As can be seen what we are talking about here is some pretty simple high school level maths that will tell you so much about what the car is doing. I realise I have repeated this example, but if your serious about being a competitive race engineer, it's about reaching a standard and not pandering to the lowest common denominator. What we have discussed here is what I would expect from a junior data/assistant race engineer. If you don't want to learn how to do this other professions beckon.

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The last example I'd like to discuss is techniques to estimate initial Traction circle radius. The common myth here is people will tell you this is far too complicated and should be handled by professionals. Well you about to find out for yourself how straight forward it can be.

Let's consider cars for a typical GT class sportscar. The particulars for this particular car are,

Table – 3 – Typical GT car parameters

Item	Value
Total Mass (m_t)	1330 kg
$C_L A$	2.4
Front weight distribution (wdf)	0.43 (43% on front axle)
Max a_y	2.4g
Max speed at Max a_y	220 km/h
Roll Distribution at front	0.5
Mean track (tm)	1.65m
Centre of gravity height	0.335m
Air Density (p)	1.225 kg/m^3

For clarity's seek let me refer to the roll distribution at the front as RLD_f. To keep this discussion simple I'm going to assume two things. Firstly the aero distribution will be the same as the weight distribution. I know this is rarely the case but I'm putting it here to simplify things. I'm also going to assume a symmetric setup. When we walk through the methodology it will be pretty clear how we can extend this approach for the asymmetric case. It's also a wise approach to slightly over estimate your Max lateral acceleration and the speed this occurs at. It just gives you a bit more flexibility. Also g is acceleration due to gravity, which is 9.8 m/s^2.

The first point of this discussion is to estimate the maximum tyre loads. This will be given by equation 2,

$$\begin{aligned} L_{MF} &= \frac{wdf}{2}\cdot\left(m_t\cdot g + \tfrac{1}{2}\rho V^2 C_L A\right) + \frac{RLD_f\cdot m_t\cdot a_y\cdot g\cdot h}{tm} \\ L_{MR} &= \frac{(1-wdf)}{2}\cdot\left(m_t\cdot g + \tfrac{1}{2}\rho V^2 C_L A\right) + \frac{(1-RLD_f)\cdot m_t\cdot a_y\cdot g\cdot h}{tm} \end{aligned} \tag{3}$$

Saving the reader the arithmetic our max loads are 7157.7 N at the front and 8376.8 N at the rear. As a further factor of safety I'm going to multiply these loads by 20%. This is going to cover us if we decide to go really crazy with downforce. Consequently our final tyre load estimation will be,

TABLE 4: Max Tyre Loads

Tyre Load	Value
Front	8590 N
Rear	10052 N

To get us going for the Max tyre force curve I'm going to assume a function of Load only. The function we are going to fit is the following,

$$F_{\max} = k_a \cdot (1 - k_b \cdot L)L$$
$$k_b = \frac{1}{2 \cdot L_{MAX}} \qquad (4)$$

Where L is the load of the tyre in N and F_{max} represents the traction circle ellipse in N. This curve is a simple parabolic fit to ensure we get max tyre force at the specified peak load and k_a represents the initial coefficient of friction with no load applied to the tyre. Going on from our values in Table 3 the k_b values for our tyre model are,

k_b	Value
Front	5.82×10^{-5}
Rear	4.97×10^{-5}

Now we have this we can now estimate the k_a values. By using equation (4) and applying a force equilibrium for the front axle it can be seen that,

$$wdf \cdot m_t \cdot a_y \cdot g = k_a\left((1 - k_b \cdot L_{out})L_{out} + (1 - k_b \cdot L_{in})L_{in}\right)$$
$$L_{OUT} = \frac{wdf}{2} \cdot \left(m_t \cdot g + \tfrac{1}{2}\rho V^2 C_L A\right) + \frac{RLD_f \cdot m_t \cdot a_y \cdot g \cdot h}{tm} \qquad (5)$$
$$L_{IN} = \frac{wdf}{2} \cdot \left(m_t \cdot g + \tfrac{1}{2}\rho V^2 C_L A\right) - \frac{RLD_f \cdot m_t \cdot a_y \cdot g \cdot h}{tm}$$

Similarly for the rear we see,

$$(1 - wdf) \cdot m_t \cdot a_y \cdot g = k_a\left((1 - k_b \cdot L_{out})L_{out} + (1 - k_b \cdot L_{in})L_{in}\right)$$
$$L_{OUT} = \frac{(1 - wdf)}{2} \cdot \left(m_t \cdot g + \tfrac{1}{2}\rho V^2 C_L A\right) + \frac{(1 - RLD_f) \cdot m_t \cdot a_y \cdot g \cdot h}{tm} \qquad (6)$$
$$L_{IN} = \frac{(1 - wdf)}{2} \cdot \left(m_t \cdot g + \tfrac{1}{2}\rho V^2 C_L A\right) - \frac{(1 - RLD_f) \cdot m_t \cdot a_y \cdot g \cdot h}{tm}$$

Doing the arithmetic and solving for k_a it is seen that,

k_a	Value
Front	2.72
Rear	2.56

What we have just done is that by using some basic equations and working for the arithmetic we have come up with an initial estimate of a tyre model. While this is not perfect you can see by using hand calcs we've started to inform ourselves a great deal about potentially what these tyres are capable of. Also for the interested readers don't take the numbers here I've presented at face value. Use table 3 and work through equations (3) – (6) and prove the results to yourselves. It's actually not that hard you just have to pop the numbers in and it should be pretty obvious where this all goes.

The case studies we have discussed all well and good, but what happens when you get something that's a bit out of the ordinary? Believe it or not it's actually not as hard as you think and the road map was given to me by my high school Physics teacher, Phil Bailey a very long time ago. The road map is the following,

1. State all the known variables.
2. State the unknown variables.
3. Sit back and think what class of problem is this.
4. Write out the equation.
5. Solve the equation and state the answer clearly.

This problem solving technique serves as the back bone of everything I do and is one of the most valuable lessons I have learnt. He also taught me two other things as well. Firstly memorise all the kinematic equations of motion, and work everything in SI Units. He taught me this when I was an impressionable 16 year old boy and so I did what I was told. This has served me well many years later and if you're a student reading this I would strongly suggest you do the same.

Summing up as we have seen here doing hand calculations provides some very valuable insights into what the race car is doing. While hand calculations don't tell you the complete story, it forces you to think about the problem and you learn an awful lot in the process. It also makes your time on the computer far more valuable because once you have done your basic sums you start to develop an instinct about what to expect when you do your CFD and FEA. This is invaluable because it builds up your instincts and gives you that edge you need to make quick decisions that can win or lose a race. This is why hand calculations is a must have skill for any serious race engineer.

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