

Deriving a Tyre Model from Nothing.

One of the most encouraging things I have seen over the past couple of months have been a number of articles on tyre modelling and elementary simulation techniques. Some of these articles have been extremely informative, but others have made me think oh dear. However a recurring theme that came to me in all of these articles is are there some simple techniques for constructing a tyre model from scratch.

The purpose of this article is too show you some simple techniques to come up with a first cut for a tyre model. By using some simple hand calculations and a bit of simulation we can go along way to filling in quite a few of the blanks. I'm doing this for two reasons. Firstly to provide you with the skills so that you can sanity check information that you get, whether this is from a tyre manufacturer or an article. Secondly for those of you with lap time simulation software this will provide an excellent start point that you can use to refine your simulations.

Let me state from the outset that I am unashamedly using ChassisSim for the basis of our work here. It's what I'm most familiar with, and I have no desire to re-invent the wheel. I'm also going to be discussing some temperature effects as well and in my humble opinion the new thermo mechanical tyre model in ChassisSim v3 gives us some very useful insights into the wonderful world of grip, load and temperature.

However as always let me always add this important caveat. The perfect race car tyre model doesn't exist. If someone claims it does exist then the said individual needs to be shot for his or her own safety!

The basis of this tyre model that we will be constructing will have the following form as outlined in equation 1,

$$F_y = fn(\alpha) \cdot fn(L_T, T_T) \quad (1)$$

Here α is slip angle and L_T is the load on the tyre and T_T is the surface temperature of the tyre. To get the longitudinal forces α is replaced by Slip ratio SR. Obviously there are other factors that correct with camber and traction ellipse characteristics, but I haven't shown these because I want to distil the tyre model to it's absolute raw essentials so that you know what is going on underneath the hood.

The first term in this equation is $fn(\alpha)$, which is bounded between -1 and 1 and not surprisingly is 0 when α is 0. The most common representation of this is the Pacjeka curve, which I have illustrated in Fig-1 for positive slip angle.

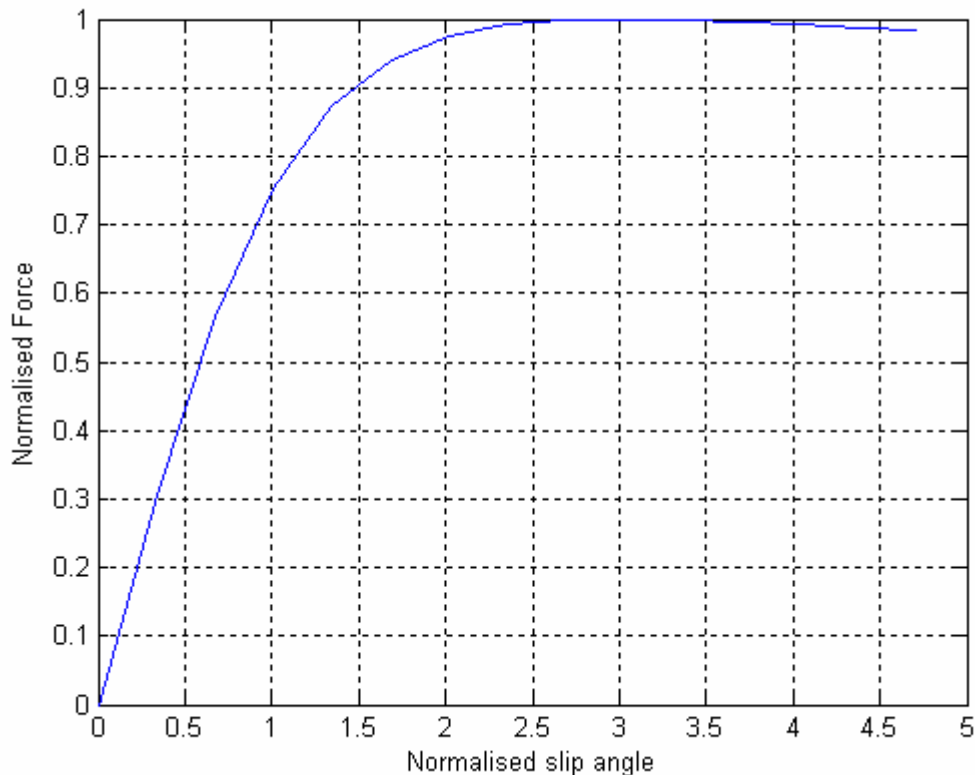


Fig-1- Plot of $f_n(\alpha)$ for positive slip ratio.

This is the curve fit that we all know and love and are familiar with. The only caveat I'll add with this is notice how as the slip angle increases, $f_n(\alpha)$ drops down to 0.9. In reality if that was totally true then on paper understeer and oversteer shouldn't be such a big deal. I would invite someone more versed in the Pacjeka model to give a further explanation of this. This is something to keep in the back of your mind. Also as a rough rule of thumb our peak of 1 should occur for slip angles of about 6 – 7 degrees or slip ratios of about 8 – 10%. The differences are the exceptions that prove the rule.

The function of $f_n(L_T, T_T)$ represents the maximum possible force the tyre can exert for a given tyre load and temperature. This effectively represents the traction circle radius of the tyre, and as far as I am concerned this is the centre piece of the tyre model. I consider this to be the case because from all this all other things flow. The determination of this curve will be the focus of the article and what we are after will be illustrated in Fig – 2.

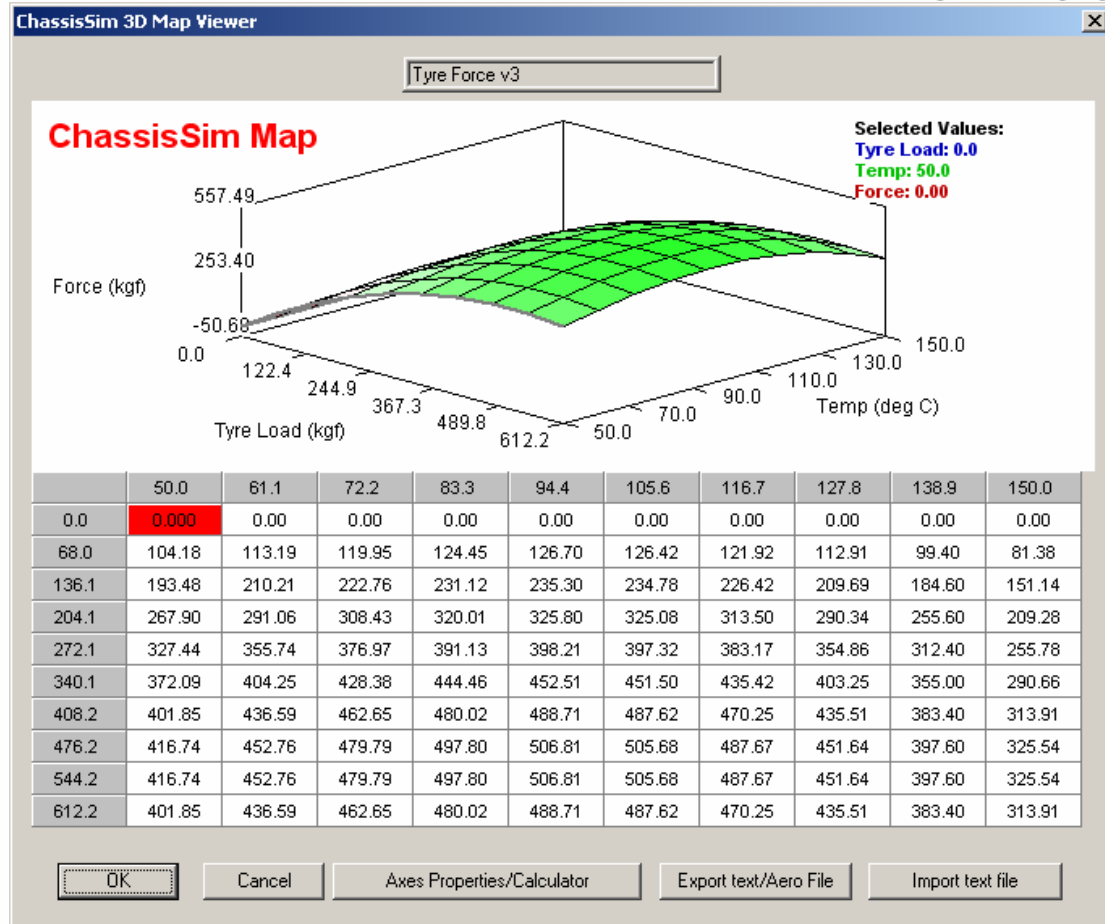


Fig-2 – Traction circle radius as a function of Load and Temperature.

To help our discussion here let's consider a case study of a FIA GT2 sportscar. The particulars for this particular car are,

Item	Value
Total Mass (m_t)	1330 kg
C_{LA}	2.4
Front weight distribution (wdf)	0.43 (43% on front axle)
Max a_y	2.4g
Max speed at Max a_y	220 km/h
Roll Distribution at front	0.5
Mean track (t_m)	1.65m
Centre of gravity height	0.335m
Air Density (p)	1.225 kg/m ³

For clarity's seek let me refer to the roll distribution at the front as RLD_f . To keep this discussion simple I'm going to assume two things. Firstly the aero distribution will be the same as the weight distribution. I know this is rarely the case but I'm putting it here to simplify things. I'm also going to assume a symmetric setup. When we walk through the methodology it will be pretty clear how

we can extend this approach for the asymmetric case. It's also a wise approach to slightly over estimate your Max lateral acceleration and the speed this occurs at. It just gives you a bit more flexibility. Also g is acceleration due to gravity, which is 9.8 m/s^2 .

The first point of this discussion is to estimate the maximum tyre loads. This will be given by equation 2,

$$\begin{aligned} L_{MF} &= \frac{wdf}{2} \cdot \left(m_t \cdot g + \frac{1}{2} \rho V^2 C_L A \right) + \frac{RLD_f \cdot m_t \cdot a_y \cdot g \cdot h}{tm} \\ L_{MR} &= \frac{(1-wdf)}{2} \cdot \left(m_t \cdot g + \frac{1}{2} \rho V^2 C_L A \right) + \frac{(1-RLD_f) \cdot m_t \cdot a_y \cdot g \cdot h}{tm} \end{aligned} \quad (2)$$

Saving the reader the arithmetic our max loads are 7157.7 N at the front and 8376.8 N at the rear. As a further factor of safety I'm going to multiply these loads by 20%. This is going to cover us if we decide to go really crazy with downforce. Consequently our final tyre load estimation will be,

TABLE 1: Max Tyre Loads

Tyre Load	Value
Front	8590 N
Rear	10052 N

To get us going for the Max tyre force curve I'm going to assume a function of Load only. For those of you anxious about temperature, don't worry we'll cover this a little bit later. To quote Mr Myagi from the Karate Kid, "First learn stand then learn fly". The function we are going to fit is the following,

$$\begin{aligned} F_{\max} &= k_a \cdot (1 - k_b \cdot L) L \\ k_b &= \frac{1}{2 \cdot L_{\max}} \end{aligned} \quad (3)$$

Where L is the load of the tyre in N and F_{\max} represents the traction circle ellipse in N. This curve is a simple parabolic fit to ensure we get max tyre force at the specified peak load and k_a represents the initial coefficient of friction with no load applied to the tyre. Going on from our values in Table 1 the k_b values for our tyre model are,

k_b	Value
Front	5.82×10^{-5}
Rear	4.97×10^{-5}

Now we have this we can now estimate the k_a values. By using equation (3) and applying a force equilibrium for the front axle it can be seen that,

$$wdf \cdot m_t \cdot a_y \cdot g = k_a ((1 - k_b \cdot L_{out})L_{out} + (1 - k_b \cdot L_{in})L_{in})$$

$$L_{OUT} = \frac{wdf}{2} \cdot (m_t \cdot g + \frac{1}{2} \rho V^2 C_L A) + \frac{RLD_f \cdot m_t \cdot a_y \cdot g \cdot h}{tm} \quad (4)$$

$$L_{IN} = \frac{wdf}{2} \cdot (m_t \cdot g + \frac{1}{2} \rho V^2 C_L A) - \frac{RLD_f \cdot m_t \cdot a_y \cdot g \cdot h}{tm}$$

Similarly for the rear we see,

$$(1 - wdf) \cdot m_t \cdot a_y \cdot g = k_a ((1 - k_b \cdot L_{out})L_{out} + (1 - k_b \cdot L_{in})L_{in})$$

$$L_{OUT} = \frac{(1 - wdf)}{2} \cdot (m_t \cdot g + \frac{1}{2} \rho V^2 C_L A) + \frac{(1 - RLD_f) \cdot m_t \cdot a_y \cdot g \cdot h}{tm} \quad (5)$$

$$L_{IN} = \frac{(1 - wdf)}{2} \cdot (m_t \cdot g + \frac{1}{2} \rho V^2 C_L A) - \frac{(1 - RLD_f) \cdot m_t \cdot a_y \cdot g \cdot h}{tm}$$

Doing the arithmetic and solving for k_a it is seen that,

k_a	Value
Front	2.72
Rear	2.56

At this point you now have enough data to use the lap time simulation package to run some provisional simulations. Let's look at the initial simulation for these numbers. This is shown in Fig-3,

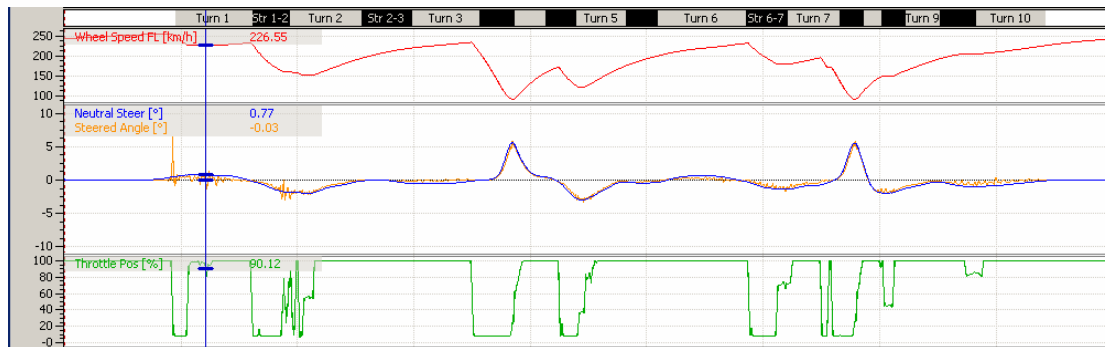


Fig-3 – Initial Simulation of base numbers,

What I have shown here is speed, Steering and neutral steer and throttle. There are two things that are apparent here. In turn 1 and Turn 2 we should be expecting speeds of 210 km/h and 120 km/h respectively, and we are way over on speeds. This is to be expected because we have over specified the tyre. Also the neutral steer line is almost on top of the steering line. This is no accident because equations (4) and (5) have virtually dictated a neutral steer line since the forces are exactly balanced.

So consequently what we need to do is we need to drop k_a for the front and the rear, and we need to reduce it more at the front to encourage understeer. My final k_a values are shown in Table 3,

Table 3 – Start and finish k_a and k_b

	k_a Start	K_b Finish
Front	2.72	1.71
Rear	2.56	1.97

This gave us corner speeds of 207 km/h and 128 km/h in Turn 1 and 2. Now we are a bit closer. It only took as 4 or 5 simulations to get to this point. We would have got there a lot faster if I had been more specific on our initial conditions, but I did this deliberately to show you what to do when things didn't work out as advertised.

Now that we are now in the baseline for tyre forces let's now discuss modelling tyre temperatures. Before I discuss in detail the tyre temperature model let me outline the strategy we are pursuing here. One of the nice things about ChassisSim v3 is that one of the returned logged variables is tyre temperature, so you can actually play with the variables to dial in the characteristics you want. Also because we are using a 2D tyre model (That is tyre force as a function of Load only) we can get the tyre temperature characteristics sorted out first before worrying about incorporating the temperature into tyre forces. This way we incrementally build the tyre model.

To get going, the best approach is to actually model the carcass and surface temperature into one unit. I realise this is cutting corners a bit but we do need some approximations to get us going. Our tyre temp governing equations can be approximated by,

$$\rho \cdot c_p \frac{dT}{dt} = H.F \cdot \sqrt{(F_y \alpha V_T)^2 + (F_x SR V_T)^2} - \kappa(T_t - T_{amb}) - \kappa_{track}(T_t - T_{track}) \quad (6)$$

In this equation we have,

- P = Tyre density (kg/m³) (Approximately 900 for rubber)
- c_p = Tyre specific heat (Approximately 2 for rubber)
- H.F = Heat factor multiplier. Correction factor to dial in tyre heat input.
- F_y = Lateral tyre force (N).
- F_x = Longitudinal tyre force (N).
- V_T = Velocity of the tyre (m/s)
- A = Slip angle (radians)
- SR = Slip ratio
- K = Thermal conductivity to air.
- K_{track} = Thermal conductivity to track.

Modelling tyre temperatures and its idiosyncrasies is an article in itself , but there are some party tricks you can use.

- The HF/ pc_p control the peak tyre temperature.
- The thermal conductivity controls the decay. The higher the more the less the temperature will bleed off and vice versa.

What we are looking for is an increase in temperatures of 40 – 50 deg C from the rest temperature going down the straight. As much as I would like to say there is a nice hand calculation you can perform at present this is something you need to dial in using simulation. This is why we are using the 2D load model first. But this really does illustrate the power of simulation. Armed with a rough idea of what to expect we can actually dial in what to expect. Is this perfect or ideal. Absolutely not. However what it does do is it sanity checks other information you get. When logging temperature you should get something that looks like Fig – 4.

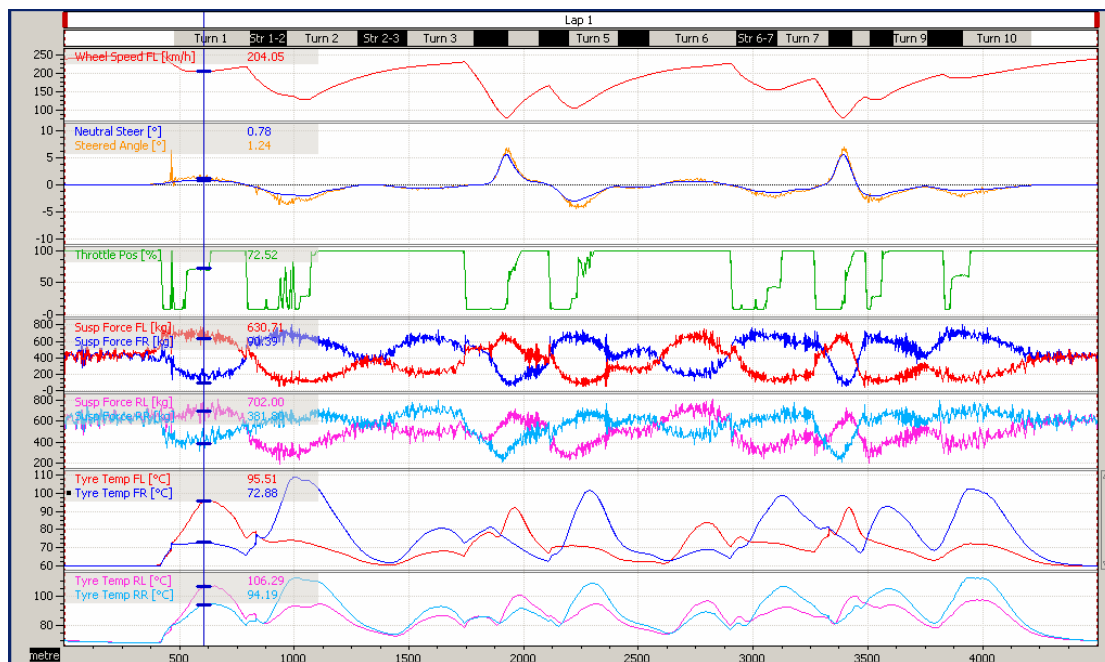


Fig-4 A plot of simulated tyre temperatures.

The last step in this process is to know incorporate tyre temperature into the Tyre force model. This is where things get really interesting and this is where we hit a fork in the road. If you're just after a simple model that you can use for quick and dirty calculations then you could stop at this point. For those of you who really want to keep going then read on.

The approach we are going to use is to multiply our Max Tyre force function by a temperature function scaled between 0 and 1. Mathematically the tyre force function will look something like this,

$$F_{\max} = fn(L) \cdot fn(T) \quad (7)$$

The real question we need to ask is what function do we choose. Let's consider this graphically and this is shown in Fig-5.

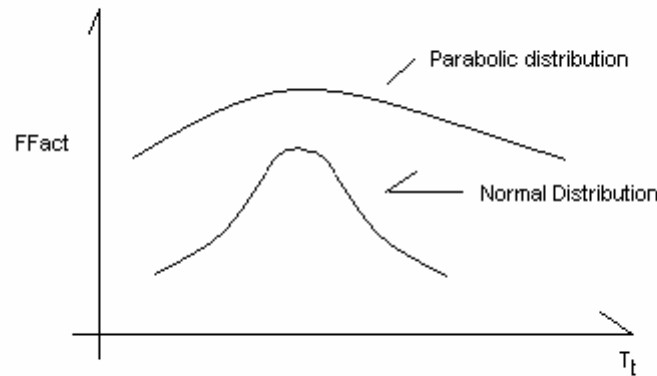


Fig-5 – Different temperature functions.

The first function shape is the parabolic shape. This shape best suits a tyre with gradual temperature characteristics. Typically this is a tyre when it isn't all that critical to be in the temperature envelope. Mathematically this function has the following equation,

$$FFact = 1 - k \left(\frac{T - T_{OPT}}{T_{OPT}} \right)^2 \quad (8)$$

Where T is the tyre temperature and T_{OPT} is the optimum tyre temperature. Since I am a stickler for SI units I use Kelvin for the temperature scale.

The second function is the normal characteristic. This represents when the tyre has only a small temperature range in which to work. Handling symptoms is when the car seems to understeer then suddenly snaps to oversteer. This function shape for this function is shown in Equation (9),

$$FFact = e^{-\left(\frac{T - T_{OPT}}{2\pi T_{DEV}} \right)^2} \quad (9)$$

Here T_{DEV} represents the standard deviation of temperature that we decay the function at.

The best way to choose what the temperature should be doing is to look at some simulated data. Let's have a look at Fig-6,

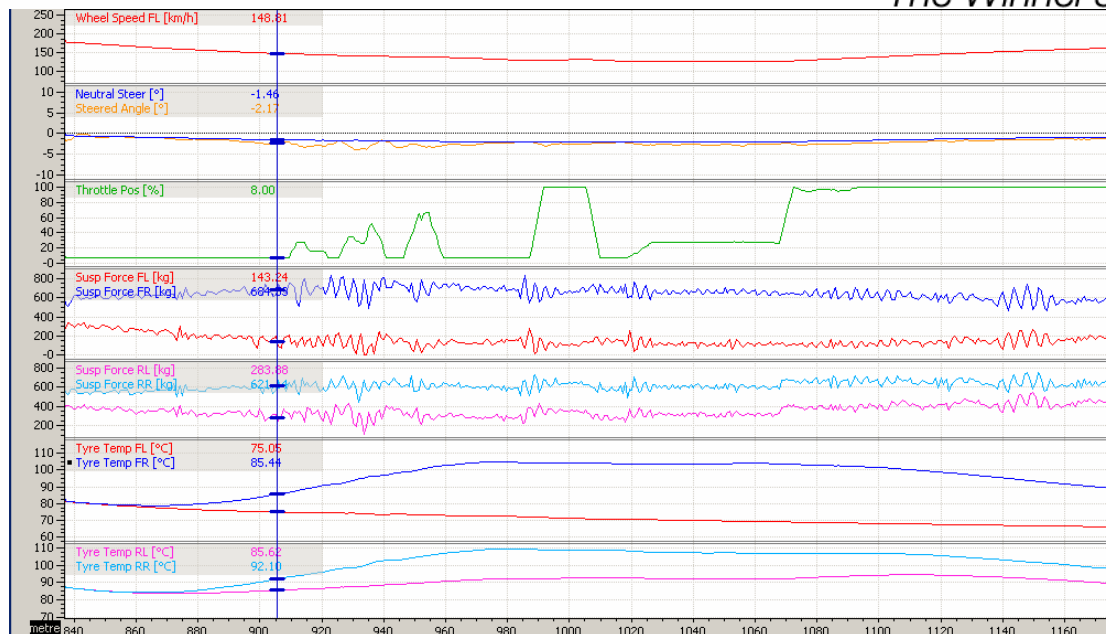


Fig-6 A plot of Temperature and steer in a low speed corner.

What we have done here is we have setup the front tyre model to deliberately provoke turn in understeer followed by a snap to oversteer. On turn in the front tyre temperature is 85 deg C and it Mid corner it peaks at 105 deg C. In this case we choose a normal distribution to deliberately provoke this situation. The Force factor for temperature in this case was 0.7 at 85 deg C and we peaked at 1 at 100 deg C. While I'll be the first to admit this is a bit obvious it illustrates how to use the simulated data.

To draw all these elements together what we are doing is that we are using some basic hand calculations, to get us in the vicinity of what we expect and then we use simulation to refine the tyre model to get it to do what we want. Obviously what is critical is comparing our simulated data to actual data to ensure we are not being lead astray.

Is what we are doing capable of producing a tyre model to be used in the heat of battle. The answer is no and yes. This tyre model is something you wouldn't use in the heat of battle. But it gets us close enough that we can use to do some rough approximations, so we can see what is going on with the car and we need something to do quick and dirty simulations with. However this model puts as close enough, where tools such as the ChassisSim tyre force optimisation toolbox can work to their full potential.

In concluding then tyre modelling isn't something that is reserved for the lucky few. Rather by using some basic hand calculations, simulation and a lot of commonsense we can get something that puts us in the ball park. Will this give you the perfect tyre model? No it won't. However it will give you valuable insights on what your race tyre is doing. If you can do this then you are well ahead of the pack.