The Damper Workbook

Over the last couple of months a number of readers and colleagues have been talking to me and asking questions about damping. In particular what has been cropping up has been the mechanics of calculating wheel rates, and damping ratios. This for me has been very exciting because it shows me that people have been genuinely thinking about this and that's great. However what it also shows me is that people have been missing some intermediate steps on how to calculate and use damping ratios and wheel rates and this is what we'll be addressing in this article.

Let me state from the outset that I'm going to be reviewing a number of matters I have discussed before in previous articles. Forgive me if this sounds like repetition but from time to time it's actually a good thing to review some important fundamentals.

To kick off this discussion we first need to get our heads around about what the difference is between a spring rate and a wheel rate. To illustrate let's consider a spring/damper unit with a bell crank. This is illustrated in Fig-1

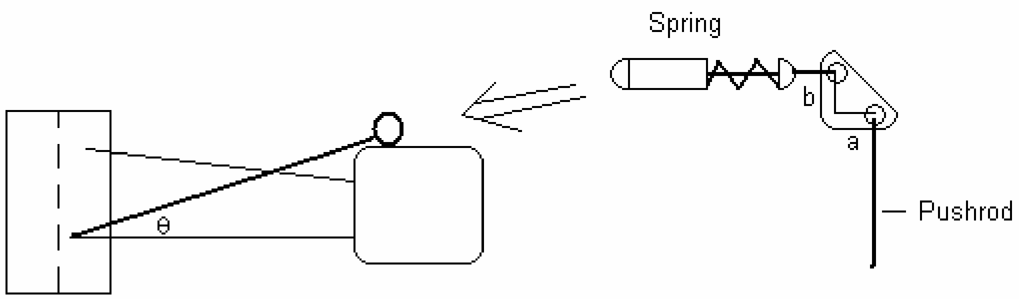

Fig – 1 – An illustration of a spring damper unit with a bell crank.

The reason I have illustrated with a bell crank is to illustrate the difference in the way between the wheel moves and the way the damper moves. This is what is termed motion ratio and this is the thing that separates the spring rate from the wheel rate. The spring rate is the rate of the spring and damper measured at the damper. The wheel rate is the rate the tyre is going to see and this is dictated by the motion ratio and spring rate.

Let us know discuss the mechanics of how to calculate the wheel rate.

To calculate wheel rate we first need to determine the motion ratio. To keep this discussion simple we'll assume linear motion ratios and springs. We'll discuss non linear motion ratios a little bit

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later. The motion ratio defines the relationship of damper to wheel movement and we can define it as,

$$MR = \frac{\partial Damper}{\partial Wheel} \qquad (1)$$

Equation (1) tells us that the motion ratio is simply the slope of damper movement vs wheel movement. So if the wheel changes by say 15mm and the damper moves by 11.25mm the motion ratio is,

$$MR = \frac{\partial Damper}{\partial Wheel} = \frac{11.25}{15} = 0.75$$

There will be some who will define the motion ratio as wheel and damper movement. I have always preferred to do it as damper on wheel because it gives me a direct measure of the forces acting on the wheel.

Now that we know motion ratios the wheel rate can be readily calculated. The wheel rate is given by,

$$WR = MR^2 * SR \qquad (2)$$

The terms of the equation here are,

WR = Wheel rate
MR = Motion ratio specified on damper on wheel.
SR = The spring rate at the damper.

Let's walk through an example of how to do this. Let's say we have a spring of say rate 140 N/mm (about 800 lbf/in) with a motion ratio of 0.75. So the wheel rate is given by,

$$\begin{aligned} WR &= MR^2 * SR \\ &= 0.75 * 0.75 * 140 \\ &= 78.75 N/mm \end{aligned}$$

Remember everything we do in this business is driven by working the tyre. What the wheel rate tells us is the spring rate the tyre is seeing. This is why it's so important to calculate this properly and while this might seem trivial this is a vitally important skill.

The next step is knowing how to read damper force curve correctly. To illustrate this let's consider a typical force vs peak velocity curve which is shown in Fig – 2

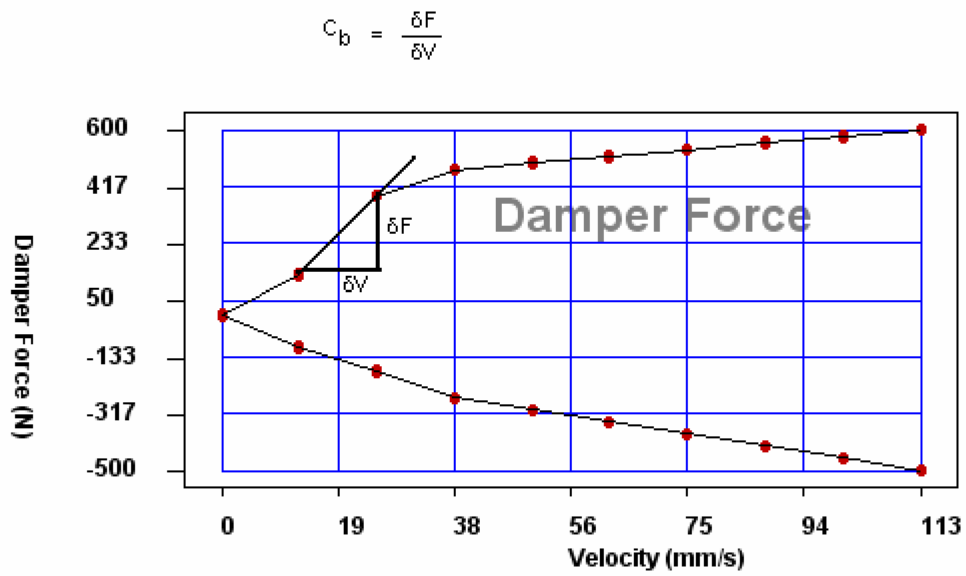

Fig-2 – A peak force vs peak velocity curve for a damper.

You can be forgiven a multitude of sins in damping provided you read this right. The critical thing is to calculate the damping rate or the slope of this curve. This is the thing that counts in damping. So the calculation of damping rate is given by,

$$C = \frac{\partial Force}{\partial Velocity} \quad (3)$$

Here C is the damping rate, dForce is the change in damping force and dVelocity is the change in damping velocity. There are a couple of traps that I need to alert you on. First things first, be tight on your units, so Forces in N, and velocity in m/s. The damping rate unit is N/m/s. You'll hate me now but you'll thank me later. The second thing is calculate this moving forward on the damping curve not backwards.

That all being said using Fig-2 let's to do an example calculation and we'll discuss what scale of numbers to expect. So looking at Fig-2 where we have illustrated our slope, the damping rate at this point is given by,

$$\begin{aligned} C &= \frac{\partial Force}{\partial Velocity} \\ &= \frac{410 - 141.5}{(25-10)*10^{-3}} \\ &= 17900 N/m/s \end{aligned}$$

I realise these numbers are a bit approximate but what counts is the way in which we did it. So we marched it forward and calculated the slope of the curve. That's all there is to it. The only trick we

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did here was to multiply the damper readings by 10^{-3} which converts mm to m. Apart from that it's pretty simple stuff. In terms of some rough rules of thumbs, rates in excess of 15000 N/m/s usually are representative of low speed damping when we want body control. Rates of 2000 – 5000 N/m/s apply to high speed damping when we want to filter bumps out.

You might be thinking this is all well and good but how do we tie this into a wheel rate? The answer is really simple. Remember equation (2) to convert spring rate to wheel rate. It's exactly the same for damper rate. This is given by,

$$C_{WHEEL} = MR^2 * C_{DAMP} \qquad (4)$$

Here we have,

C_{WHEEL} = Damping rate the wheel sees
C_{DAMP} = Damping rate at the spring damper unit
MR = Motion ratio

So for our example with a Motion ratio of 0.75 the wheel damping rate is given by,

$$C_{WHEEL} = MR^2 * C_{DAMP} = 0.75 * 0.75 * 17900 = 10070 N/m/s$$

Now that we know how to calculate both wheel spring rates and damping rates we can now calculate damping ratios using the quarter car approximation. Remember the quarter car approximation is a very powerful tool to estimate what our damping rates should be and the spring rates we should be considering. To refresh everyone's memory our quarter car approximation looks like this,

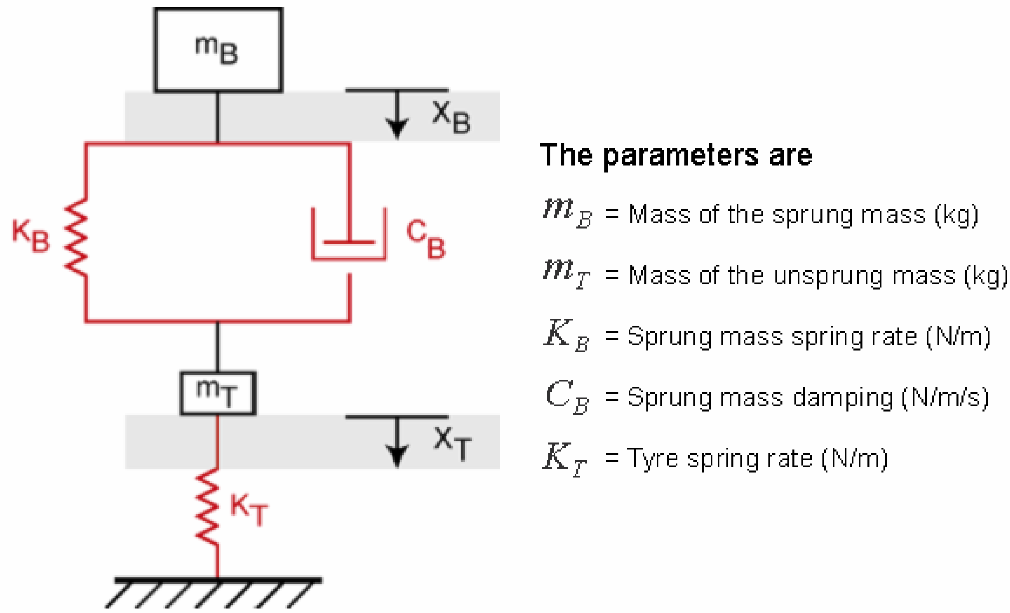

Fig-3 – Quarter car approximation

The trick here is to visualise the spring/damper unit at each corner of the car. While it isn't obviously the full story of what's going on with the car it's a valuable building block to quantify the spring and damping characteristics of the race car. Mathematically the crux of the quarter car method is the following,

$$\omega_0 = \sqrt{\frac{K_B}{m_B}} \tag{5}$$

$$C_B = 2 \cdot \omega_0 \cdot m_B \cdot \zeta$$

$$\zeta = \frac{C_B}{2 \cdot \omega_0 \cdot m_B} \tag{6}$$

Here the terms of the equation are,

K_b	= Wheel rate of the spring (N/m)
C_b	= Wheel damping rate of the spring (N/m/s)
m_b	= Mass of the quarter car.
ω_0	= Natural frequency (rad/s)
ζ	= Damping ratio

The power of the quarter car is that given a damping ratio we want we can readily calculate the damping rate we want. Once we know the damping rates we want we can then turn around to a damper builder and see this is the damping curve we want. This is why this technique is so powerful.

So using the spring and damping rates we discussed earlier let's do a worked example. When we calculate this ensure our spring rates are in N/m and our masses are in kg. I realise this causes considerable consternation to some of my friends in North America, but you'll hate me know and thank me later. Some rough rules of thumbs to converting to N/m.

- If the spring rate is in lbf/in multiply by 175.126.
- If the rate is in N/mm multiply by 1000.

So for our example here let's assume a quarter car mass of 125kg. Crunching the numbers we see,

$$K_B = MR^2 \cdot SR = 0.75*0.75*140*1000 = 78750$$

$$C_B = MR^2 \cdot C_{DAMP} == 0.75*0.75*17900 = 10070$$

$$\omega_0 = \sqrt{\frac{K_B}{m_b}} = \sqrt{\frac{78750}{125}} = 25.1 rad/s$$

$$\zeta = \frac{C_B}{2 \cdot \omega_0 \cdot m_b} = \frac{10070}{2*25.1*125} = 1.6$$

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At this point you might be thinking this might be great but how do we use this? Where we use this is understanding what the damping ratio is telling us. Again this is going over some old material but recall our damping guide,

Table – 1 – Rough outline to damping ratios

Damping Ratio Range	What this applies to
0.3 – 0.4	Ideal for filtering out bumps
0.5 – 1.0	This deals with body control.
1.0 +	This deals with extreme body control/driving temperature into the tyres.

The damping ratio is telling what effect you want from your spring damper unit. Remember high values of damping ratio tell you we want to control the body. Low values tell us we are trying to filter out bumps and/or keep the wheel in contact with the ground. Remember when you calculate the damping ratios you're effectively looking at the damper's finger print.

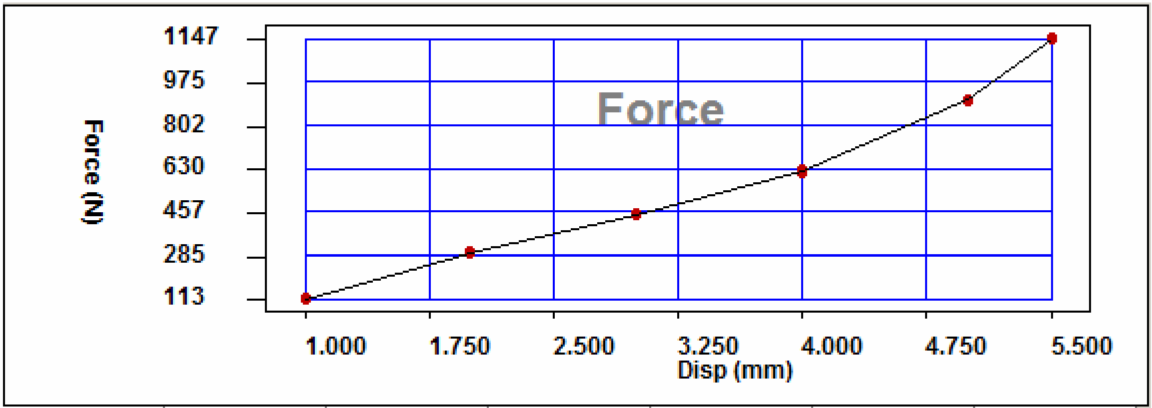
Also too the damping ratio will vary throughout the velocity range of the damper. This is a consequence of what we discussed when calculating damper rates. Remember damper rates will affect the damping ratio and this drives the behaviour of the damper. To illustrate this why don't we consider an example I presented a number of articles a go where I calculated the damping ratios through out the velocity range of Fig-2. In this case the motion ratio was 1, and the spring rate was 175 N/mm or a 1000 lbf/in and the quarter car mass was 157kg. The results are shown in Table – 2

Table-2 – Damping ratios for Damper presented in Fig-2

Velocity (mm/s)	Damping ratio in bump	Damping ratio in rebound
0	1.24	0.95
13	2.03	0.6
25	0.616	0.707
38	0.175	0.31
50	0.167	0.286
63	0.174	0.31

Table 2 presents some enlightening insights into what this damper is trying to do. First things first the damping ratios from 0 tell me immediately this is a high downforce car. The high damping ratios are tell tale signs this is a high downforce car. The high damping ratios immediately suggest that body control is paramount. Looking at the bump at 13mm/s the damping ratio jumps to 2.03. This indicates the damper engineer is trying to give some feel to the car as well as load the tyres. Beyond this range the dampers blow off to a low ratio to allow the car to ride the bumps. In rebound from 13 – 25 mm/s the damping ratio is 0.7. This tells me body control is still paramount. Beyond the damping ratios blow off to 0.3. This tells me this is designed for the bumps. I encourage the reader to look at Fig-2, and using the example we have presented to rework these numbers. Hint – when calculating slopes in rebound use absolute values.

The next question that must be addressed is how to deal with non linear springs and motion ratios. Again it's actually a lot easier than you think. First let's deal with non linear springs. Where you will encounter non linear springs is using bump rubbers. All this means is the spring rate changes. That's it and to calculate is really easy. Let's consider this bump rubber that is illustrated in Fig – 4



disp	1.000	2.000	3.000	4.000	5.000	5.500
Force	112.700	294.000	445.900	617.400	901.600	1147.000

Fig-4 – Calculating spring rate of a non linear spring.

Let's say this is a bump rubber and it is compressed by 4mm. Let's also say the base spring rate is 140 N/mm. The spring rate of the bump rubber is,

$$SR_{BR} = \frac{901.6 - 617.4}{5 - 4} = 284.2 N/mm$$

So the actual spring rate at the damper is,

$$\begin{aligned} SR &= SR_{BASE} + SR_{BR} \\ &= 140 + 284.2 \\ &= 424.2 N/mm \end{aligned}$$

I realize this is a bit of a trivial example but nonetheless illustrates how straightforward this is. All we need to do is march forward on the lookup table, calculate the slope and that's how we get our spring rate. It goes without saying if this was the only active spring rate we wouldn't need to add the other spring component.

We also deal with non linear motion ratios in a very similar way. Remember at its core all a motion ratio is measuring the slope of damper movement over wheel movement. Once you get your head around that you can calculate for any given situation. Let's consider this non linear motion ratio characteristic that is presented in Fig 5

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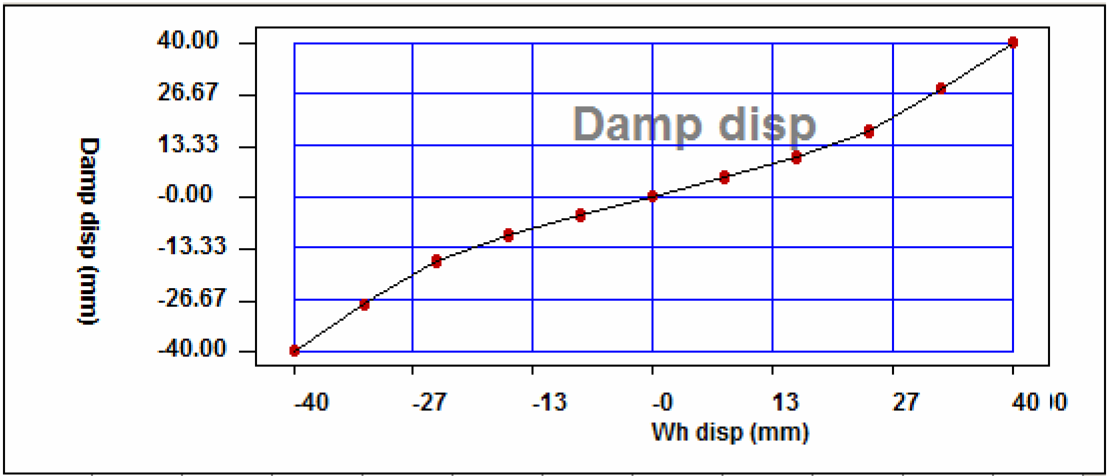

Wh disp	-40.00	-32.00	-24.00	-16.00	-8.00	0.00	8.00	16.00	24.00	32.00	40.00
Damp disp	-40.00	-28.00	-17.00	-10.00	-5.00	0.00	5.00	10.00	17.00	28.00	40.00

Fig-5 – Non linear motion ratio

So let's just say we want to calculate the motion ratio at 10mm of damper displacement. Just like with the spring and damper all we need to do is to calculate the forward rate. The calculation for this is,

$$MR = \frac{17-10}{24-16} = 0.875$$

It is as simple as that. In terms of units to calculate you can choose what ever units you want. Just be consistent when dividing.

The implications of these non linearities are that when we encounter them, the damping ratios are going to change. There is no need to be nervous about this. This just reflects the physical reality is the wheel rates or damping rates have changed and we need to deal with it. However at least you have the language to describe what has happened.

In closing then I trust this resolves the methodology of how to calculate wheel and damping rates and it fills in the blanks on how to calculate damping ratios. As we have seen this isn't hard, you just need to fill in the blanks. The next step I leave to you the reader. It's time for you to back into your setup information and calculate this, and produce your own version of Table 2 for various setups. Trust me the results will be well worth it.

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