CHASSISSIM
The Winner's Edge

TYRE LOAD ANALYSIS

One of the oldest quotes in Motor racing is from Mark Donohue,
"The footprints of your tyres are all that lie between you and St Peter". An obvious corollary of this is that it is a very good idea to know what your tyre loads and how they are distributed around the race car.

The purpose of this article is to take an in depth look at how the tyre loads for the race car can be deduced. To do this we will be using a simplified model of the race car and delving into the equations of motion to work out what is actually going. Be warned this article will be using a lot of maths and for this I make no apologies. Mathematics is the language of the physical sciences, and to try and do this with out maths, is a little bit writing this article in Hieroglyphics.

To begin this analysis lets consider a simplified illustration of the race car with the forces applied to it. I'll call this the beam-pogo stick approximation of the race car,

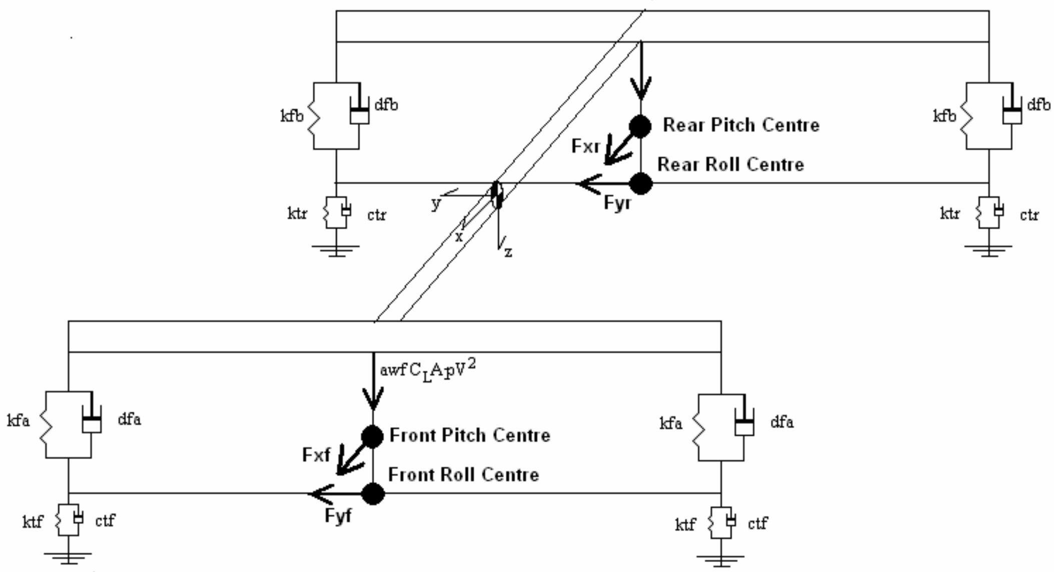

Fig-1: Beam Pogo Stick approximation of the race car.

For the reader's reference the earth symbols beneath the bottom of the tyre springs represent the ground. For simplicity I have omitted roll bar springs and third springs. I am also assuming an infinitely stiff chassis. Hence the use of the beams connecting the car bits together. The beauty of the beam pogo stick approximation is that it allows us to quickly visualize what is going on with the forces that are applied to it. For the purposes of this article, all spring, damper and bar rates are quoted in wheel rates. That is it is assumed the springs, bars and dampers are all connected to the wheel.

ABN 3517 202 936 9
ChassisSim Technologies™
9 Harriet Street Marrickville
NSW 2204 Australia
Danny Nowlan, Director mobile: +61 425 219 375

The major forces that are applied to the race car are,
*The downforce of the car given by Faero_f at the front and Faero_r at the rear.
*The weight of the car $m_s g$ and the unsprung masses.
*The front Lateral force Fyf applied to the front roll centre and the pitch force Fxf applied through the front pitch centre.
*The rear Lateral force Fyr applied to the rear roll centre and the pitch force Fxr that is applied through the rear pitch centre.
For the reader's clarification the pitch centres front and rear are the anti dive or anti squat of the car. For the roll forces the interested reader might ask what about roll centre migration on the x-axis of the car. We are assuming force based roll centres that apply a moment arm equivalent to the combined forces multiplied by the distance between the roll centre and the centre of gravity height.

Before continuing this analysis we need to clarify that all motions of a rigidly connected car take place about the centre of gravity. The only exceptions to this occur when we have chassis's that suffer from significant structural flexure. The point at which this will effect our calculations is when we through the chassis in the bin! One of the most common misconceptions I see is the concept of the mass line that lies on the car centre line. I'm sorry but it doesn't exist. All vehicle motion takes place as a direct result of forces and moments about the centre of gravity. If you have a problem with this I suggest you take it up with either Issac Newton or a higher power.

Before we begin on the equations of motions lets sort out some nomenclature. These are,

kfa	spring rate or function for the front main spring.
dfa	damper rate or function for the front damper.
ktf	spring rate of the front tyre.
ctf	damping rate of the front tyre.
kfb	spring rate or function for the rear main spring.
dfb	damper rate or function for the rear damper.
ktr	spring rate of the rear tyre.
ctr	damping rate of the rear tyre.
krbf	front roll bar rate.
kthsf	third spring rate or function at the front.
dthsf	third spring damping force or function at the front.
krbf	rear roll bar rate.
kthsr	third spring rate or function at the rear.
dthsr	third spring damping force or function at the rear.

The first step in formulating the differential equations is to consider the wheel movements. The wheel movements will be defined as;

ABN 3517 202 936 9
ChassisSim Technologies™
9 Harriet Street Marrickville
NSW 2204 Australia
Danny Nowlan, Director mobile: +61 425 219 375

ma	movement of the spring/damper on the left front wheel.
mb	movement of the spring/damper on the right front wheel.
mc	movement of the spring/damper on the rear left wheel.
md	movement of the spring/damper on the rear right wheel.
mrbf	movement of the roll bar at the front.
mthsf	movement of the third spring at the front.
mrbr	movement of the rear roll bar.
mthsr	movement of the third spring at the rear.

The wheel movements are given by the equations below;

$$ma = z - a \cdot \theta - \frac{tf \cdot \phi}{2} + y_1 \quad (1)$$

$$mb = z - a \cdot \theta + \frac{tf \cdot \phi}{2} + y_2 \quad (2)$$

$$mrbf = \frac{ma - mb}{2} \quad (3)$$

$$mthsf = \frac{ma + mb}{2} \quad (4)$$

$$mc = z + b \cdot \theta - \frac{tr \cdot \phi}{2} + y_3 \quad (5)$$

$$md = z + b \cdot \theta + \frac{tr \cdot \phi}{2} + y_4 \quad (6)$$

$$mrbr = \frac{mc - md}{2} \quad (7)$$

$$mthsr = \frac{mc + md}{2} \quad (8)$$

where;

y_1 to y_4	wheel movements of tyres 1 to 4 respectively.
z	downward displacement of the sprung mass.
θ	pitch angle of the sprung mass.
ϕ	roll angle of the sprung mass.

Knowing what the wheel movements are, the spring and damper forces may be determined. The differential equations are;

$$m_S z'' = Faero\_f + Faero\_r + m_s g - kfa(ma) - kfa(mb) - dfa(ma') - dfa(mb') - kthsf(mthsf) - dthsf(mthsf') - kfb(mc) - kfb(md) - dfb(mc') - dfb(md') - kthsr(mthsr) - dthsr(mthsr') \quad (9)$$

$$I_y\theta'' = a\cdot(kfa(ma) + kfa(mb) + dfa(ma') + dfa(mb') + kthsf(mthsf) + dthsf(mthsf')) - b\cdot(kfb(mc) + kfb(md) + dfb(mc') + dfb(md') + kthsr(mthsr) + dthsr(mthsr')) + b\cdot Faero\_f - a\cdot Faero\_r + F_{XF}\cdot(h - pc_F) + F_{XR}\cdot(h - pc_R) \quad (10)$$

$$I_x\phi'' = 0.5\cdot tf\cdot(kfa(ma) - kfa(mb) + dfa(ma') - dfa(mb') + 2\cdot krbf\cdot mrbf) + 0.5\cdot tr\cdot(kfb(mc) - kfb(md) + dfb(mc') - dfb(md') + 2\cdot krbf\cdot mrbr) - F_{YF}\, rc_F - F_{YR}\, rc_R \quad (11)$$

$$m_{tf}\, y_1'' = ktf(z_1 - y_1) + ctf(z_1' - y_1') - (kfa(ma) + dfa(ma') + 0.5(kthsf(mthsf) + dthsf(mthsf')) - frbf) \quad (12)$$
$$m_{tf}\, y_2'' = ktf(z_2 - y_2) + ctf(z_2' - y_2') - (kfa(mb) + dfa(mb') + 0.5(kthsf(mthsf) + dthsf(mthsf')) + frbf) \quad (13)$$
$$m_{tr}\, y_3'' = ktr(z_3 - y_3) + ctr(z_3' - y_3') - (kfb(mc) + dfb(mc') + 0.5(kthsr(mthsr) + dthsr(mthsr')) - rrbf) \quad (14)$$
$$m_{tr}\, y_4'' = ktr(z_4 - y_4) + ctr(z_4' - y_4') - (kfb(md) + dfb(md') + 0.5(kthsr(mthsr) + dthsr(mthsr')) + rrbf) \quad (15)$$

where $'$ and $''$ denote the first and second derivatives respectively and;

z_1 to z_4	ground displacements of tyres 1 to 4 respectively.
m_S	mass of the sprung mass.
I_X	Moment of inertia of the sprung mass about the x-axis.
I_Y	Moment of inertia of the sprung mass about the y-axis.
m_{tf}	mass of the front tyre.
m_{tr}	mass of the rear tyre.
frbf	front roll bar force = krbf.mrbf
rrbf	rear roll bar force = krbr.mrbr

The first thing the reader might ask is why have we gone to all this trouble? Well the reason for this is to give the reader the complete set of tools to really evaluate what is going on. Also this gives the interested reader the opportunity to model these using packages such as Matlab or Maple. I should also add for clarity I haven't put in the pitch and roll centre effects for the unsprung mass. These are simply added into the differential equations for the unsprung masses. I'd refer you to my book the Dynamics of the RaceCar for the details.

The first thing that pops out of Figure 1 and these equations is what the tyre loads are. A Free body diagram of each tyre in contact with the ground, along with equations (12) to (15) show that the tyre Loads are,

$$L_1 = ktf(z_1 - y_1) + ctf(z_1' - y_1') \quad (16)$$
$$L_2 = ktf(z_2 - y_2) + ctf(z_2' - y_2') \quad (17)$$
$$L_3 = ktr(z_3 - y_3) + ctr(z_3' - y_3') \quad (18)$$
$$L_4 = ktr(z_4 - y_4) + ctr(z_4' - y_4') \quad (19)$$

ABN 3517 202 936 9
ChassisSim Technologies™
9 Harriet Street Marrickville
NSW 2204 Australia
Danny Nowlan, Director mobile: +61 425 219 375

To be clear on the conventions L_1 is the left front, L_2 is the right front and L_3 is the left rear and L_4 is the right rear. I leave the derivation of equations (16) to (19) to the reader, but think about the situation physically. If you push a flexible object into the ground, the load your putting on to the ground is a direct function of the force you need to compress said object. Newton's third law of motion, for every action there's an equal and opposite reaction.

To pull it back a level and draw some quantitative conclusions lets assume the following,

*All spring and damper rates are linear.

*Pre load values are extremely low.

*Let's resolve it for a static condition.

I'm not assuming this to fudge the results; I'm assuming this so we can gain some insights into the mechanics of load transfer and where the numbers go. Solving equations (9) to (15) for the static condition, that is the derivatives equal to 0 and what we discussed in equations (16) to (19) we can derive some very interesting results. Let's consider a lateral load to the car applied exactly as per the weight distribution. Also to make things easier lets assume the front and rear tracks are the same. Assuming the car is in a steady state velocity situation the roll distribution factor can be deduced by the following,

```
rcm   = rcf + wdr*(rcr - rcf);                    (17)
hsm   = h - rcm;                                  (18)
  rsf = (krbf + kfa)*ktf/( kfa + krbf + ktf);     (19)
  rsr = (kfb + krbr)*ktr/(kfb + krbr + ktr);      (20)
 prm  = rsf/(rsr + rsf);                          (21)
  prr = (wdf*rcf + prm*hsm)/h;                    (22)
```

where;

rcm	mean roll centre (measured in metres).
rcf	front roll centre height (measured in metres).
rcr	rear roll centre height (measured in metres).
wdr	weight distribution at the rear of the car.
wdf	weight distribution at the front of the car.
h	Centre of gravity height of the car (measured in metres).
rsf	wheel spring rate in roll for the front (Newtons/metre).
rsr	wheel spring rate in roll for the rear (Newtons/metre).
ktf	front tire spring rate (Newtons/metre).
ktr	rear tire spring rate (Newtons/metre).
kfb	spring rate of the rear coil, acting at the wheel (Newtons/metre).
krbr	rear roll bar rate (Newtons/metre).
prm	Lateral load transfer due through the sprung mass.
prr	lateral load transfer distribution at the front.
tm	mean track of the vehicle.

To get to this we simply solve equations (9 – 15) for the state variables and simply plug the results into equations (16 – 19).

ABN 3517 202 936 9
ChassisSim Technologies™
9 Harriet Street Marrickville
NSW 2204 Australia
Danny Nowlan, Director mobile: +61 425 219 375

Despite our rather simplified assumption equation (22) contains mountain information of how Lateral load transfer distribution works. They tell us the following,
*The roll centre is one of the quickest ways you can vary your weight transfer front and rear. An example of this is that the rear roll centre is one of the biggest setup variables on a V8 Supercar.
*The limit of your roll spring is the tyre spring rate.
*Following on from equation (22) low speed damping front and rear has a very similar effect as the spring rate. Consequently equation (22) provides a very good visualisation of the effects that it can have.

However I encourage the interested reader, don't take my word for it, build a Matlab or Maple model of equations (9) to (15) and throw a plethora of Lateral inputs at it. You'll get to appreciate how setup variables effect the load distribution and it will answer a lot of questions about what does what. This is also the power of dynamic simulation programs such as ChassisSim. Due to the dynamic nature of its simulation it allows the race engineer to investigate how every setup variable effect each bit of the load transfer giving the race engineer valuable information on what the car is doing at every aspect of the turn.

Another corollary of equations (9) to (15) is how vital it is to characterise the aero loads of the race car. Using equation 22 another way of expressing tyre loads is the following,

$L_1 = (wdf*m_t*g + Faero\_f)/2 + prr*(Fyf + Fyr*)h/tm + \text{other terms}$ (23)

$L_2 = (wdf*m_t*g + Faero\_f)/2 - prr*(Fyf + Fyr*)h/tm + \text{other terms}$ (24)

$L_3 = (wdr*m_t*g + Faero\_r)/2 + (1 - prr)* *(Fyf + Fyr*)h/tm + \text{other terms}$ (25)

$L_4 = (wdr*m_t*g + Faero\_r)/2 - (1 - prr)* *(Fyf + Fyr*)h/tm + \text{other terms}$ (26)

It doesn't take a rocket scientist to work out as the level of downforce increases then Faero_f and Faero_r will start to have a pretty big effect on what the car will and will not do. I am continually astounded/shocked at the amount of race/data engineers who still do not characterise the aeromap of the car. For those of you who don't have laser sensors don't use this as an excuse. An examination of Figure 1 and the use of math channels because provides an invaluable approximation of the aeromap. In particular for some classes of racing I have worked in, constructing the aeromap this way was quite literally the difference between being in the top 5 and languishing in mid field. I could say more on this issue but I will let equations (23) to (26) speak for themselves.

In closing then this article has focused on giving the reader the tools they need to work out what the tyre loads of the race car are actually doing. Race car engineering is a tough business with very few hard and fast rules, which makes it even more important to appreciate the fundamentals of what makes the car work. Understanding what effects your tyre loads is of vital importance for tuning the race car. If all that I have down is to give you the tools you need to further your understanding of this, then I have succeeded.

ABN 3517 202 936 9
ChassisSim Technologies™
9 Harriet Street Marrickville
NSW 2204 Australia
Danny Nowlan, Director mobile: +61 425 219 375

ChassisSim

The Winner's Edge

ABN 3517 202 936 9
ChassisSim Technologies™
9 Harriet Street Marrickville
NSW 2204 Australia

Danny Nowlan, Director mobile: +61 425 219 375