# The Vehicle Dynamics of rallying.

For the last two months what has been dominating my radar screen has been adapting ChassisSim for WRC. The tarmac bit was easy but the tough bit has been adapting ChassisSim to run on dirt and ice. This has been a job that has been challenging yet incredibly informative all at the same time. The challenging bit has been resolving why you have to run well into the post stalled region of the tyre and then resolving how to stay there. This is what we'll be discussing in this article.

Let me state from the get go right now I am not pretending to be an expert on this. If truth be told I'm actually writing this article more for me than you at this point so I can start to get some things straight in my head. That being said I've learnt a lot on the way so if you are involved with rallying or have any interest as to what happens when a car goes sideways and then read on. Hopefully we can all learn something in the process.

So here is the question why do you want to go sideways in a rally car? For all of us that have been involved in tarmac/circuit racing this is a cardinal sin. It looks incredibly impressive but when it comes to tarmac/open wheel racing we all know it's a guaranteed way to kill your speed. The answer to this question lies in what the tyre is doing.

The answer as to why we want to go sideways on dirt and ice comes down to the slip characteristics of the tyres. To really hammer home the point let's illustrate this graphically. A typical force vs slip angle characteristic for a road racing tyre is shown below,

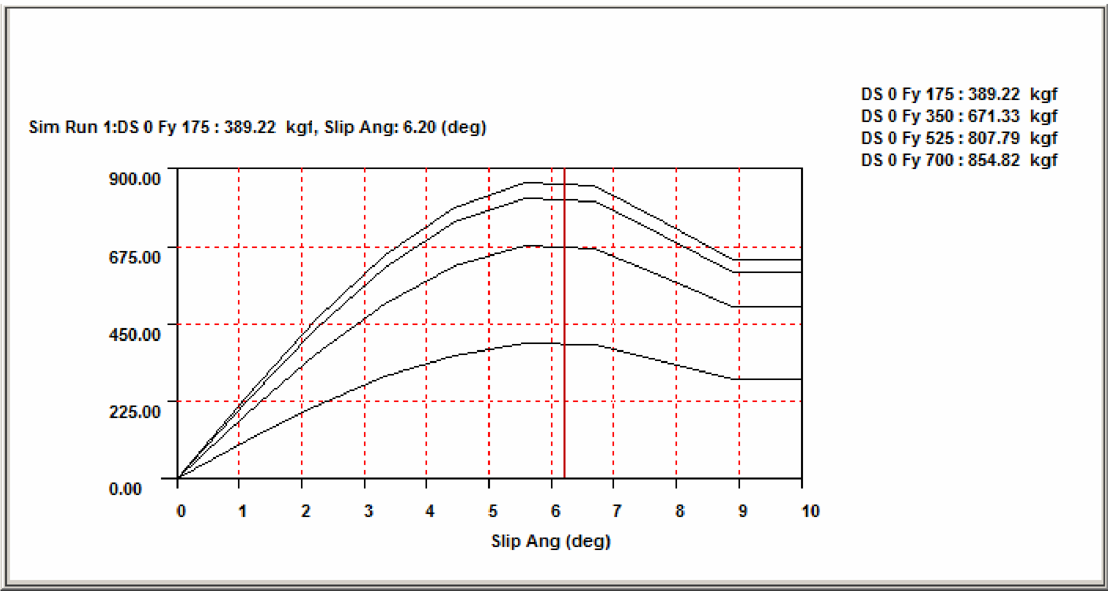


**Fig - 1 - Lateral force vs slip angle characteristic for a Road tyre.**

ABN 3517 202 936 9
ChassisSim Technologies™
9 Harriet Street Marrickville
NSW 2204 Australia
**Danny Nowlan**, Director mobile: +61 425 219 375

The thing to note in this curve is how significantly the grip drops away after you have exceeded the peak slip angle. In the post stalled region this is in the order of 10 - 20%. Consequently if you want to go fast on a road racing tyre there is no point being sideways because the grip isn't there.

When you are on dirt and ice the tyre characteristics are a totally different ball game. When I was doing my literature search I came across two excellent thesis. These where Michael Croft-White's thesis from Cranfield University "Measurement and Analysis of Rally Car Dynamics at High Attitude Angles" and a Thesis from Stanford university entitled - "Dynamics and control of drifting in automobiles" by Rami Yusef Hindiyeh. The upshot from both of these thesis is that when you are post stalled the grip doesn't drop off that much. In particular White's thesis did some basic tyre modelling from a beta sensor he had developed. This is presented in Fig - 2

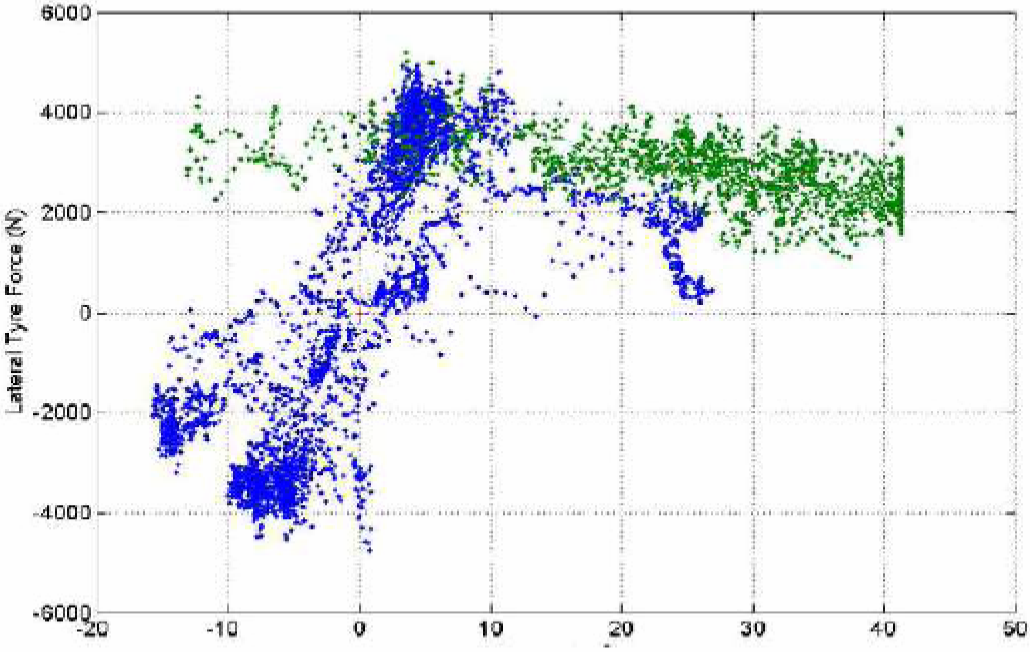


**Fig - 2 - Force vs slip angle characteristics of a rally tyre**

The key to note is what is happening in the post stalled region. Looking at slip angles well in excess of 20 deg the grip has only dropped of by 10%. This is significant because there is grip to be had in the post stalled region.

If the drop in post slip grip is mild the reason there is grip is because of what happens with the longitudinal forces of the tyre at large slip angles. This is illustrated in Fig-3 along with the equations of motion with the car,

ABN 3517 202 936 9
ChassisSim Technologies™
9 Harriet Street Marrickville
NSW 2204 Australia
**Danny Nowlan**, Director mobile: +61 425 219 375

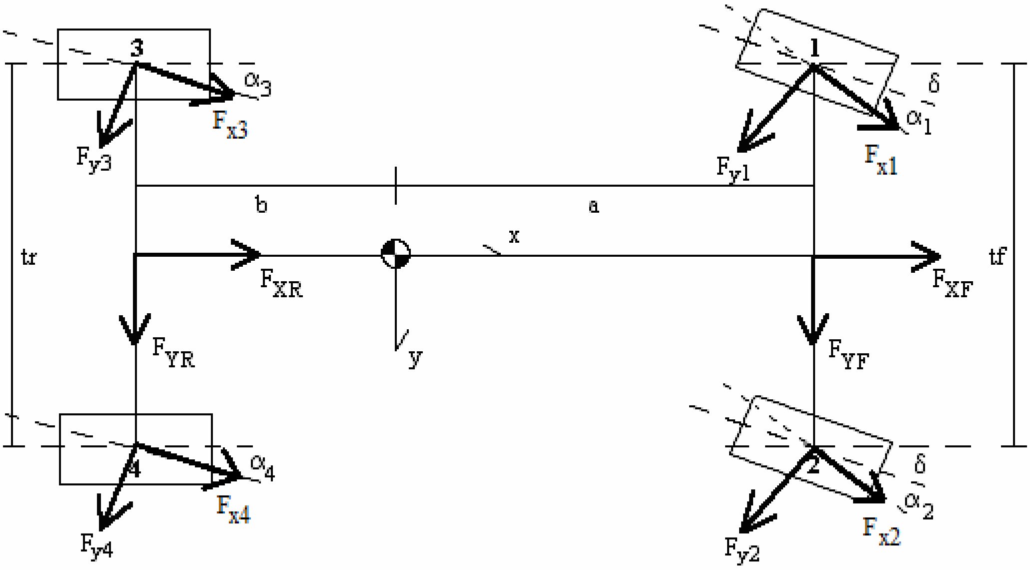


**Fig-3 - Free Body Diagram of the forces acting on the race car.**

The symbols in Figure 1 are;

$F_{y1}$ to $F_{y4}$ Lateral forces of tyres 1 to 4 respectively.

$F_{x1}$ to $F_{x4}$ Longitudinal forces of tyres 1 to 4 respectively

$\alpha_1$ to $\alpha_4$ slip angles of tyres 1 to 4 respectively.

$\delta$ steer angle of the front wheels (equal steer angles are used on both sides to keep the representation simple).

$F_{YF}$ Lateral force applied at the front axle.

$F_{YR}$ Lateral force applied at the rear axle.

The tyre loads are applied vertically down for each tyre.

All tyre forces are applied along the slip angle line. $F_{XF}$ and $F_{XR}$ are the sum of all the longitudinal forces at the front and rear respectively. Longitudinally this will not have a huge impact. As we'll see shortly it has big ramifications laterally. This is particularly apparent at large slip angles.

Also just a note about small angle assumptions. Strictly speaking they are only apply to about +/- $10^0$. However for practical calculation purposes we can stretch this to about $20^0$. Let me illustrate what I mean. In radians $20^0$ is 0.349. The sin of $20^0$ is 0.342. The cosine of $20^0$ is 0.94. Yes we sacrifice a little bit of accuracy longitudinally but the sine of the angles are still very close. Consequently the equations we are about to present still workout. The other option is to include the sine and cosine terms. While it is fully accurate, the problem is you start to lose any perspective

ABN 3517 202 936 9
ChassisSim Technologies™
9 Harriet Street Marrickville
NSW 2204 Australia
**Danny Nowlan**, Director mobile: +61 425 219 375

about what the maths is telling you. Also in rallying it is rare to see a side slip greater than 30 deg. While this is not ideal we are certainly not in fantasy land.

Also to simplify things we have also lumped in the lateral forces here as well. Using small angle assumptions it may be concluded;

$$F_{YF} = F_{y1} + F_{y2} \quad (1)$$
$$F_{YR} = F_{y3} + F_{y4} \quad (2)$$

From the derivation presented in Wong[3] the slip angles are;

$$\alpha_1 = \delta - \frac{a \cdot r + V_y}{V_x + tr \cdot r} \quad (3)$$
$$\alpha_2 = \delta - \frac{a \cdot r + V_y}{V_x - tr \cdot r} \quad (4)$$
$$\alpha_3 = \frac{b \cdot r - V_y}{V_x + tr \cdot r} \quad (5)$$
$$\alpha_4 = \frac{b \cdot r - V_y}{V_x - tr \cdot r} \quad (6)$$

where;

$V_y$ sideways velocity.
$V_x$ forward velocity.
$r$ yaw rate.

Resolving forces and moments from Fig 1, the differential equations of the race car become;

$$m_t(V_x' + V_y r) = F_{XR} + F_{XF} - \sum_{i=1}^{2}(\delta + \alpha_i) \cdot F_{yi} - \sum_{i=3}^{4} \alpha_i \cdot F_{yi} - 0.5\rho V^2 C_D A \quad (7)$$

$$m_t(V_y' + V_x r) = F_{YF} + F_{YR} + (\delta + \frac{\alpha_1 + \alpha_2}{2}) \cdot F_{XF} + \left(\frac{\alpha_3 + \alpha_4}{2}\right) \cdot F_{XR} \quad (8)$$

$$I_z \cdot r' = a \cdot (F_{YF} + \left(\delta + \frac{\alpha_1 + \alpha_2}{2}\right) \cdot F_{XF}) - b \cdot (F_{YF} + \left(\frac{\alpha_3 + \alpha_4}{2}\right) \cdot F_{XR}) \quad (9)$$

Here,

$m_t$ = Total mass of the car.
$I_z$ = the rotational inertia of the car

Equations (3) – (9) describe everything on how the race car will behave. The thing to note here is the longitudinal forces. To reiterate they are applied on the slip angle line of the tyre. So at this point you might be thinking so what? The key lies in the lateral components of the longitudinal forces. We have,

$$F_{YF\_FXF} = (\delta + \frac{\alpha_1 + \alpha_2}{2}) \cdot F_{XF}$$
$$F_{YR\_FXR} = \left(\frac{\alpha_3 + \alpha_4}{2}\right) \cdot F_{XR} \quad (10)$$

Here we have $F_{YF\_FXF}$ is the lateral force at the front induced by the front longitudinal forces and $F_{YR\_FXR}$ is the lateral force induced by the rear longitudinal forces.

Where things get really interesting is what happens when the slip angles go up. Let's illustrate this with some numbers. Let's consider a typical AWD rally car that weighs in at 1300kg. Some performance numbers are illustrated below,

**Table - 1 Rally car parameters**

| Parameter | Value |
|---|---|
| Car mass | 1300kg |
| Cornering g | 1 |
| Peak slip angle - tarmac | $6^0$ |
| Peak slip angle - dirt | $16^0$ |
| $C_dA$ | 1.1 |
| Cornering speed | 108 km/h |

I realise the cornering g on dirt will be less than tarmac but let's keep these the same for the time being. I want you to get a feel for the magnitude of the numbers.
So balancing the speeds and assuming front and rear slip angles to be the same we have

$$F_{XT} = \frac{1}{2}\rho \cdot V^2 \cdot C_D A + \alpha_p \cdot m_t \cdot a_y \quad (11)$$

Here we have,

$F_{xt}$ = Total longitudinal force applied (N)
p = air density (kg/m$^3$)
V = Car speed (m/s)
$a_p$ = Peak slip angle in radians
$m_t$ = total car mass
$a_y$ = Lateral acceleration in m/s$^2$

I realise this is not strictly accurate but it is in the ball park and I'm doing this so you get a feel for the numbers. Crunching the numbers for the tarmac and dirt modes the results are presented in Table - 2

**Table - 2 - Numbers for the balanced longitudinal forces in tarmac and dirt mode.**

| Mode | $F_{XT}$ (kgf) | Lateral component (kgf) |
|---|---|---|
| Tarmac | 198kgf | 20.7 |
| Dirt | 424 kgf | 118.7 |

So in Tarmac mode we have about 20.7 kgf of lateral force produced by the applied longitudinal force. In dirt mode this jumps to 118.7 kgf. While the analysis is incredibly over simplified it rams home the rally observation that on dirt the engine force is a significant part of your corner grip.

So how do we determine that this is viable or not? Enter what I will term the Drift feasibility equation. Let's illustrate this situation graphically. This is shown in Fig -4,

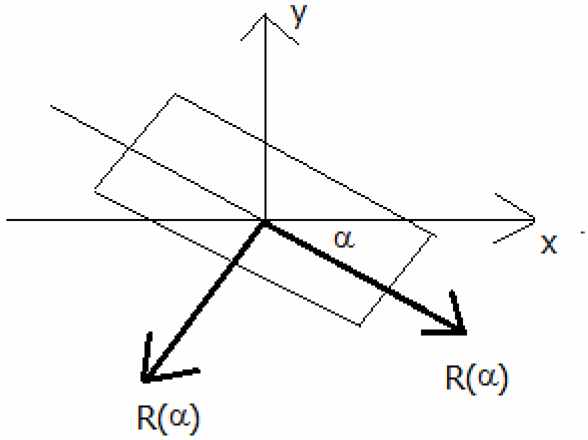


**Fig-4 Two tyre force acting both laterally and longitudinally**

As can be seen from Fig-4 we have two equal forces acting laterally and longitudinally. I will term this force R(α). Both of these components will have lateral components. Let me set R(α) such that,

$$R(\alpha) = C(\alpha)\cdot(Fm_{OUT} + Fm_{IN}) \qquad (12)$$

Here C(a) is the normalised slip curve and $Fm_{out}$ and $Fm_{IN}$ are the outer and inner traction circle radius values. Our goal here is to find the best compromise of slip angle that produces the optimum lateral grip. Our total laterally forces will be given by,

$$F_{YT} = 0.707 \cdot R(\alpha) \cdot (\cos(\alpha) + \sin(\alpha)) \qquad (13)$$

So that we are clear I am slaving the force R(α) to the force vs slip angle equation that we all know and love. However I'm still keeping it in traction circle limits so that we don't enter Fantasy Land. So the optimum slip angle will be given by deriving equation (14) as a function of slip angle. Using the product differential rule it is found that the optimum slip angle that will produce the most lateral grip will be given by,

$$\frac{\partial F_y}{\partial \alpha} = 0 = R(\alpha) \cdot (\cos(\alpha) - \sin(\alpha)) + \frac{\partial R}{\partial \alpha} \cdot (\cos(\alpha) + \sin(\alpha)) \qquad (14)$$

Equation (14) is the drift feasibility equation. This won't necessarily tell you the optimum slip angle you need to be at for drifting. However it will tell you if your tyre can actually do it. As a case in point consider Fig 5a which is a road course tyre and Fig 5b which is a rally tyre. Evaluating equation (14) for both of these curves show that Fig 5a has an optimum slip angle of $6.2^0$ and Fig 5b has an optimum slip angle of $16^0$. Also a tip for young players here. Do this numerically. List out R(α) and the subsequent derivatives. If you try and do it analytically you'll drive yourself nuts. This is the first step in seeing if it is worth your while to go sideways or not.

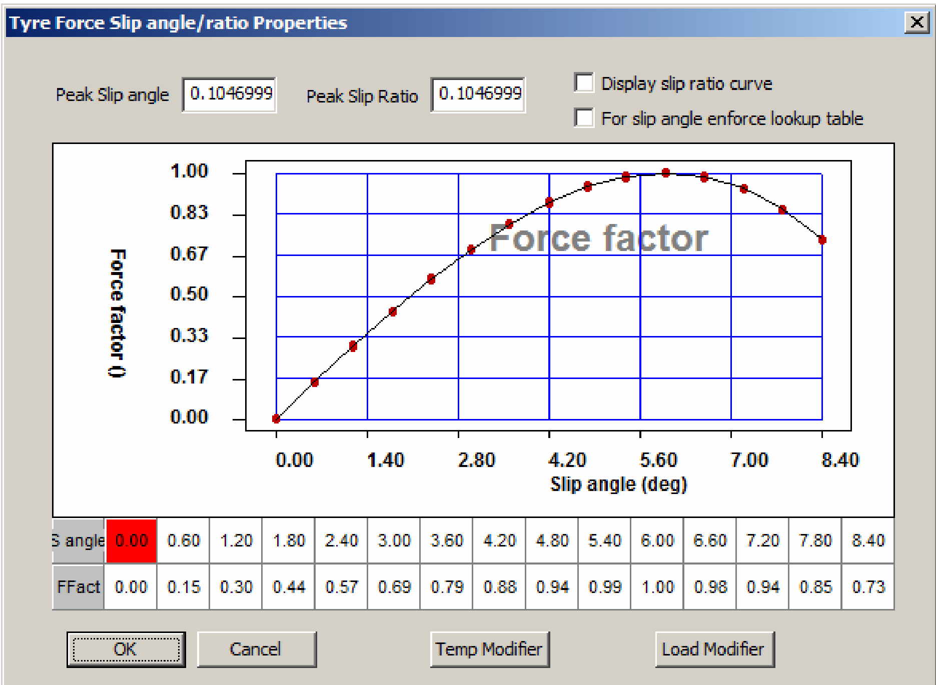


**Fig 5a - Road course tyre**

ABN 3517 202 936 9
ChassisSim Technologies™
9 Harriet Street Marrickville
NSW 2204 Australia
**Danny Nowlan**, Director mobile: +61 425 219 375

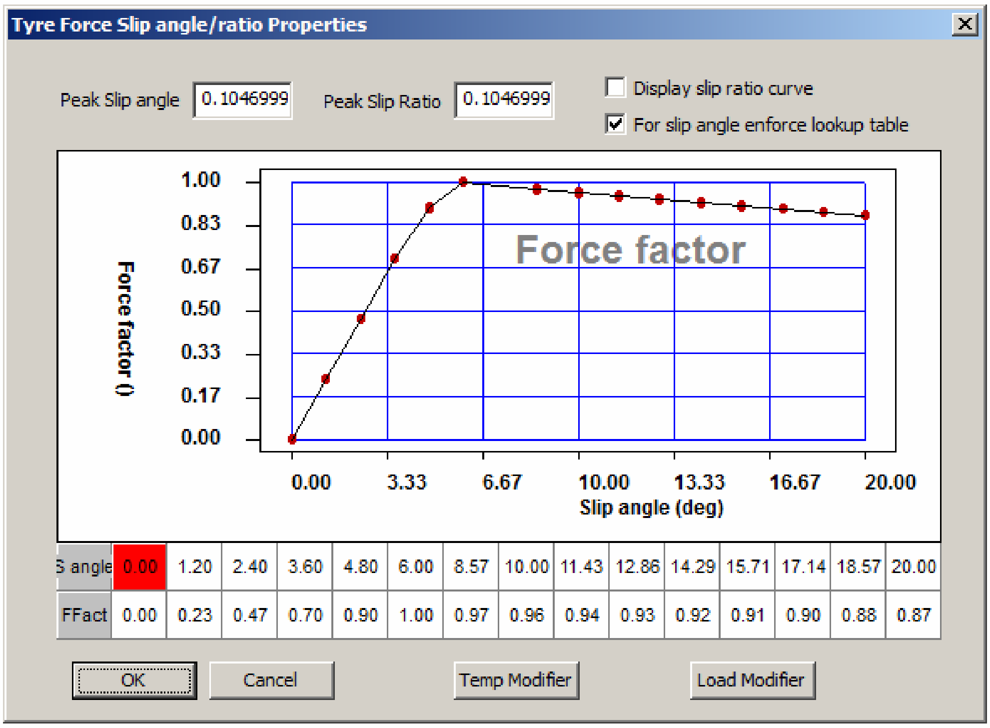


**Fig 5b - Rally tyre**

So now that we have established if it's viable or not to go sideways we know need to nail down at what angle we need to go sideways at. Remember we are drifting on dirt and ice not just because it looks impressive but we are doing this to get grip. The answer lies in the lateral grip front and rear.

Let's put some maths to this. To simplify things a little bit let's use the bicycle equations of motion for the front and rear slip angles. This is presented in equation (15)

$$\alpha_F = \delta - \frac{a \cdot r + V_y}{V_x}$$

$$\delta = \alpha_F + \frac{a \cdot r + V_y}{V_x} \qquad (15)$$

$$\alpha_R = \frac{b \cdot r - V_y}{V_x}$$

Here $\alpha_f$ and $\alpha_R$ are the front and rear slip angles. The front and rear lateral forces taking into account both the forces due to slip angle of the tyre and the longitudinal forces are,

ABN 3517 202 936 9
ChassisSim Technologies™
9 Harriet Street Marrickville
NSW 2204 Australia
**Danny Nowlan**, Director mobile: +61 425 219 375

$$F_{YF} = C_F(\alpha)\cdot(Fm_1 + Fm_2) + (\delta + \alpha_F)\cdot F_{XF}$$
$$= C_F(\alpha)\cdot(Fm_1 + Fm_2) + \left(2\cdot\alpha_F + \frac{a\cdot r + V_y}{V_x}\right)\cdot F_{XF} \qquad (16)$$
$$F_{YR} = C_R(\alpha)\cdot(Fm_3 + Fm_4) + (\alpha_R)\cdot F_{XR}$$

Let's nail down the nomenclature here. We have

$C_F(a)$ = Normalised force slip angle curve at the front
$C_R(a)$ = Normalised force slip angle curve at the Rear
$Fm_1$ = Traction circle radius at the Left front tyre for a given load.
$Fm_2$ = Traction circle radius at the Left front tyre for a given load.
$Fm_3$ = Traction circle radius at the Left front tyre for a given load.
$Fm_4$ = Traction circle radius at the Left front tyre for a given load.

Where things get really interesting is taking the derivative with respect to slip angle of equation(16). Doing the differentiation we see,

$$\frac{\partial(F_{YF})}{\partial\alpha_F} = \frac{\partial(C_F(\alpha_F))}{\partial\alpha}\cdot(Fm_1 + Fm_2) + 2\cdot F_{XF}$$
$$\frac{\partial(F_{YR})}{\partial\alpha_R} = \frac{\partial(C_R(\alpha_R))}{\partial\alpha}\cdot(Fm_3 + Fm_4) + F_{XR} \qquad (17)$$

In order to be worth your while to drift The differential of the front and rear force curves must be greater than zero. This is where the grip is and the reason the grip is there is as the slip angle increases you will actually be producing force you can use. It's the reason that you see Sprint cars on an oval hanging the tail out because that is where the grip is. If your car is rear wheel drive, the last bit of equation (17) applies. If your car is front wheel drive the first bit of equation (17) applies. If you are all wheel drive then both come into play. For rallying, equation (17) outlines the appeal of all wheel drive.

So what is the procedure to determine the slip angle that you should be drifting to satisfy equation (17). Firstly you start by choosing a corner speed and looking at the peak curvature you want to corner at. You then nominate the factor of grip you want to maintain at the rear. The crux of this is we want to maintain equilibrium both laterally and longitudinally. Keeping the slip angles the same front and rear we have the following,

$$\alpha_R \cdot F_{XR} = F_{XFR}\cdot wdr \cdot m_t \cdot V_X^{\ 2}\cdot iR \qquad (18)$$

$$F_{XR} = tsp_R \cdot\left(\frac{1}{2}\cdot\rho\cdot V^2\cdot C_D A + \alpha_R m_t \cdot V_X^{\ 2}\cdot iR\right) \qquad (19)$$

ABN 3517 202 936 9
ChassisSimTechnologies™
9 Harriet Street Marrickville
NSW 2204 Australia
**Danny Nowlan**, Director mobile: +61 425 219 375

Here $tsp_R$ is the torque split at the rear and $F_{XFR}$ is the factor of rear longitudinal tyre force we want to contribute to the lateral grip. Putting equation (19) into (18) yields the following relation for the rear slip angle we are after,

$$\alpha_R^{\,2} + \frac{0.5 \cdot \rho \cdot C_D A}{m_t \cdot iR} \cdot \alpha_R - \frac{F_{XFR}}{tsp_R} = 0 \qquad (20)$$

The solution of equation (20) will give you a reference check. You are then going to go through an iterative process to see if this makes sense. In particular if it is achievable through the slip angle curves you have. The other thing to check is the load transfer so you have the traction. The limitation will be the inside rear tyre loads. You will then check equation (17) and if all this adds up you have equilibrium. When this all checks out you have determined the rear slip angle and side slip angle you should be drifting at.

What all these equations tell you is that drifting to improve vehicle grip is only viable in low grip situations. Let's re inspect equation (17) but this time let's do it through the lens of load transfer. As a rough rule of thumb your tyre loads for a given aero load and load transfer are given by,

$$\begin{aligned}
TL_1 &= \frac{wdf \cdot m_t \cdot g}{2} + \frac{awf \cdot C_L \cdot A \cdot \tfrac{1}{2} \cdot \rho \cdot V^2}{2} + \frac{prr \cdot m_t \cdot a_y \cdot h}{tm} \\
TL_2 &= \frac{wdf \cdot m_t \cdot g}{2} + \frac{awf \cdot C_L \cdot A \cdot \tfrac{1}{2} \cdot \rho \cdot V^2}{2} - \frac{prr \cdot m_t \cdot a_y \cdot h}{tm} \\
TL_3 &= \frac{wdr \cdot m_t \cdot g}{2} + \frac{awr \cdot C_L \cdot A \cdot \tfrac{1}{2} \cdot \rho \cdot V^2}{2} + \frac{(1 - prr) \cdot m_t \cdot a_y \cdot h}{tm} \\
TL_4 &= \frac{wdr \cdot m_t \cdot g}{2} + \frac{awr \cdot C_L \cdot A \cdot \tfrac{1}{2} \cdot \rho \cdot V^2}{2} - \frac{(1 - prr) \cdot m_t \cdot a_y \cdot h}{tm}
\end{aligned} \qquad (21)$$

Where the downforce is not significant what will limit you will be the inside front and rear tyres unloading. Consequently your ability to apply the longitudinal forces you need to ensure equation (17) is greater than or equal to zero will be limited. Strictly speaking you could channel all the longitudinal force to the outside rear wheel but you will have a destabilising moment due to the tractive force trying to destabilise the car.

The last topic to touch upon is what does the racecar stability look like in the post stalled drift zone. As discussed in some of my previous articles an excellent tool to look at this which is the stability index. This can be written as,

ABN 3517 202 936 9
ChassisSim Technologies™
9 Harriet Street Marrickville
NSW 2204 Australia
**Danny Nowlan**, Director mobile: +61 425 219 375

$$SI = \frac{a \cdot \frac{\partial F_{YF}}{\partial \alpha} - b \cdot \frac{\partial F_{YR}}{\partial \alpha} - \left( a^2 \cdot \frac{\partial F_{YF}}{\partial \alpha} + b^2 \cdot \frac{\partial F_{YR}}{\partial \alpha} \right) \cdot \frac{r}{V_X}}{\left( \frac{\partial F_{YF}}{\partial \alpha} + \frac{\partial F_{YR}}{\partial \alpha} \right) \cdot wb} \qquad (22)$$

Inspecting equation (17) and putting it into equation (22) we will have still have some measure of stability. However it will be much more marginal. This is because the slope of the force vs slip angle curves are much smaller. The applied longitudinal forces are the dominant terms. The combination of equations (17) and (22) mean that if you are sliding in a rear wheel drive car you have no option but to keep the power applied. This was also confirmed in Hindiyeh(2) where his controller uses engine force as an integral part of his drift controller.

Lastly to show this isn't just theory the beginnings of this have been incorporated into ChassisSim. Find an example of a predictive rally simulation done in real time controlling the car in the post stalled zone of the tyre,

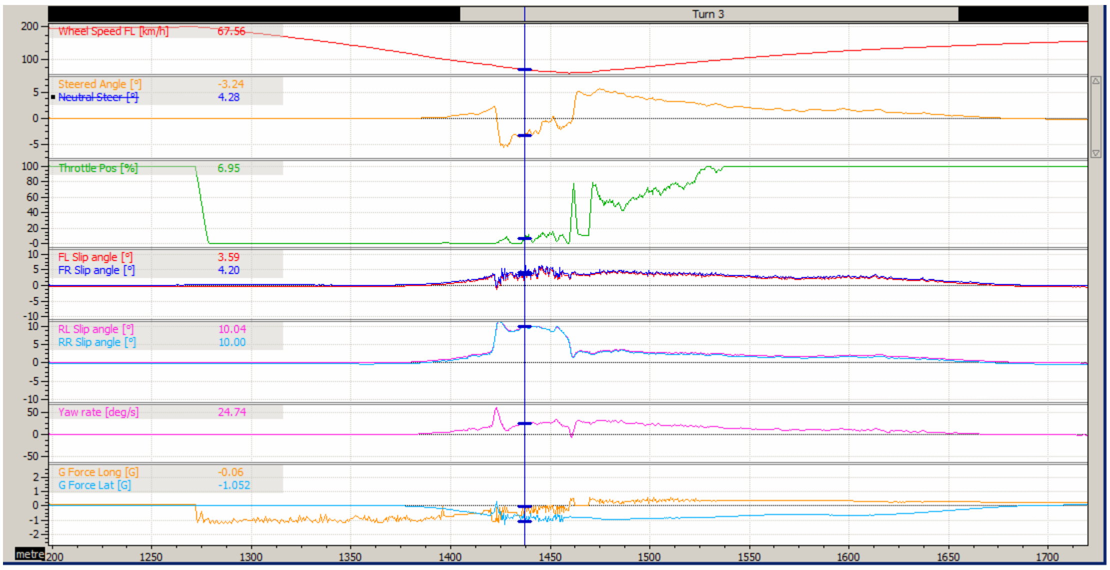


**Fig - 6: Predictive rally sim from ChassisSim controlling the car in the post stalled zone.**

The first trace is speed, the second is steered angle and the third trace is throttle. However the real traces are the fourth and fifth traces that show front and rear slip angles. The stall angle for this tyre is $6^0$ . The front slip angles are in the order of $4\text{-}5^0$. However you can see the rear slip angles are in the order of $10^0$ and they are being controlled. I should add the car model needs a bit of work and there is still some work to do. However this feature is on it's way.

In closing the vehicle dynamics of drifting are an exceptionally interesting field of vehicle dynamics. The thing that dictates why you want to drift is what happens to the tyres on dirt and ice. Here the slip angle curves drop of moderately in the post stalled region of the tyre. This makes it

ABN 3517 202 936 9
ChassisSim Technologies™
9 Harriet Street Marrickville
NSW 2204 Australia
**Danny Nowlan**, Director mobile: +61 425 219 375

viable to drift and we can readily calculate where we need to be drifting. While this is certainly not the last word on the vehicle dynamics of rallying I trust what I have given you is the mathematical framework to put some numbers to this.

## References

1) "Measurement and Analysis of Rally Car Dynamics at High Attitude Angles" - Michael Croft-White  - Cranfield University 2005
2) "Dynamics and control of drifting in automobiles" - Rami Yusef Hindiyeh - Stanford University 2013
3) J.Y Wong, *Theory of Ground Vehicles*, Wiley and sons, 1978

ABN 3517 202 936 9
ChassisSim Technologies™
9 Harriet Street Marrickville
NSW 2204 Australia
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